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Hyers-Ulam Stability of First Order Nonhomogeneous Linear Dynamic Equations on Time Scales

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Abstract: This paper deals with the Hyers-Ulam stability of the nonhomogeneous linear dynamic equation $x^{\Delta}(t) - ax(t) = f(t)$, where $a \in \mathcal{R}^+$. The main results can be regarded as a supplement of the stability results of the corresponding homogeneous linear dynamic equation obtained by Anderson and Onitsuka (Anderson D R, Onitsuka M. Hyers-Ulam stability of first-order homogeneous linear dynamic equations on time scales. *Demonstratio Math.*, 2018, **51**: 198–210).

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1 Introduction

In 1940, Ulam[1] posed the following problem concerning the stability of group homomorphisms: Under what conditions can a solution of a perturbed equation be close to a solution of the original equation. The following year, Hyers[2] solved this type of stability problem for the case of approximately additive mappings in Banach spaces. Afterwards, the result of Hyers was generalized by Rassias[3] for linear mappings by considering an unbounded Cauchy difference. Since then, there has been a great interest in the Hyers-Ulam stability (HUS) of functional and differential equations. For more detailed results, the reader can refer to these monographs (see [4]–[8] and the references therein).

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For an n-th order differential equation

$$F(t, x, x', \cdots, x^{(n)}) = 0, \qquad t \in I, \tag{1.1}$$

where I is a nonempty open interval I of $\mathbf{R}$. We say that the equation (1.1) has Hyers-Ulam stability (HUS) if and only if there is a constant $K > 0$ with the property: Given a $\varepsilon > 0$, if an n-time differentiable function $y : I \to \mathbf{R}$ satisfies

$$|F(t, y, y', \cdots, y^{(n)})| \le \varepsilon, \qquad t \in I,$$

then there exists a solution $x : I \to \mathbf{R}$ of (1.1) such that $|x(t) - y(t)| \le K\varepsilon$ for all $t \in I$. Such a constant K is called an HUS constant for the equation (1.1) on I.

In 1998, Alsina and Ger[9] proved the Hyers-Ulam stability of the differential equation

$$x' - x = 0$$

and obtained an HUS constant 3 on I. Subsequently, Miura *et al.*[10] and Takahasi *et al.*[11] considered the Hyers-Ulam stability of first order linear differential operators in a complex Banach space. As a special case, these results showed that the differential equation

$$x' - ax = 0, \qquad a \ne 0$$

has the Hyers-Ulam stability with a HUS constant $\dfrac{1}{|a|}$. Moreover, if given a solution $\phi(t)$ of the perturbed equation, then the solution $x(t)$ of the original equation satisfying

$$|\phi(t) - x(t)| \le \frac{\varepsilon}{|a|}$$

is unique. In 2017, Onitsuka and Shoji[12] further studied the Hyers-Ulam stability of the equation

$$x' - ax = 0$$

from a different perspective. Under the assumption that a differentiable function $\phi(t)$ satisfies

$$|\phi'(t) - a\phi(t)| \le \varepsilon, \qquad t \in \mathbf{R},$$

they constructed an explicit solution $x(t)$ of the corresponding equation

$$x' - ax = 0$$

satisfying

$$|\phi(t) - x(t)| \le \frac{\varepsilon}{|a|}, \qquad t \in \mathbf{R}.$$

Also, Onitsuka[13] discussed the influence of the constant step size h on Hyers-Ulam stability of the first-order homogeneous linear difference equation

$$\Delta_h x(t) - ax(t) = 0$$

on the uniformly discrete time scale $h\mathbf{Z}$. Meantime, Onitsuka[14] further investigated the Hyers-Ulam stability of the first-order nonhomogeneous linear difference equation

$$\Delta_h x(t) - ax(t) = f(t)$$

on $h\mathbf{Z}$. Recently, Anderson and Onitsuka[15] studied the Hyers-Ulam stability of the first order homogeneous linear dynamic equation

$$x^{\Delta}(t) - ax(t) = 0 \tag{1.2}$$