Communications
in
Mathematical
Research
35(2)(2019), 149–158

Volume Difference Inequalities for the Polars of Mixed Complex Projection Bodies

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Communicated by Rong Xiao-chun

Abstract: In this paper, based on the notion of mixed complex projection and generalized the recent works of other authors, we obtain some volume difference inequalities containing Brunn-Minkowski type inequality, Minkowski type inequality and Aleksandrov-Fenchel inequality for the polars of mixed complex projection bodies.

Key words: mixed complex projection body, polar, volume difference, Brunn-Minkowski type inequality, Minkowski type inequality

2010 MR subject classification: 52A20, 52A39, 52A40

Document code: A

Article ID: 1674-5647(2019)02-0149-10

DOI: 10.13447/j.1674-5647.2019.02.06

1 Introduction

The classical Brunn-Minkowski theory, that is, the theory of mixed volumes, is the core of convex geometric analysis. It originated with Minkowski[1] who combined the mixed volume with the Brunn-Minkowski inequality. The Brunn-Minkowski theory has been extended to the L_p Brunn-Minkowski theory, which combines volume and a generalized vector addition of compact convex sets introduced by Firey[2] in the early 1960s and is known as L_p addition. Lutwak initiated the new L_p Brunn-Minkowski theory in [3]–[5].

In contrast to the Brunn-Minkowski theory, in the dual theory, convex bodies are replaced by star-shaped sets, and projections onto subspaces are replaced by intersections with subspaces. Lutwak *et al.*[6]–[7] and Ludwig[8] made great effort on an Orlicz-Brunn-Minkowski theory. The theory is far more general than the L_p Brunn-Minkowski theory.

Received date: Sept. 28, 2018.
Foundation item: The NSF (11371224) of China.
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The Orlicz extension is based on the asymmetric L_p Brunn-Minkowski theory developed by Ludwig, Haberl, Schuster, Reitzner and others (see [9]–[12]). In [6]–[7], Lutwak, Yang and Zhang established two fundamental affine inequalities in the new Orlicz-Brunn-Minkowski theory, that is, the Orlicz-Busemann-Petty centroid inequality and the Orlicz Petty projection inequality.

The dual Orlicz-Brunn-Minkowski theory for star bodies were enforced firstly by Zhu *et al.*[13] and late by Gardner *et al.*[14]. They established the dual Orlicz-Minkowski inequality and the dual Orlicz-Brunn-Minkowski inequality. Moreover, Zhu *et al.*[13] introduced Orlicz intersection bodies and proposed the Orlicz-Busemann-Petty problem.

In [15], Abardia and Bernig defined the mixed complex projection and established some geometric inequalities. We follow ideas of Abardia and Bernig, obtain some volume difference inequalities including the Brunn-Minkowski type inequality, Minkowski type inequality and Aleksandrov-Fenchel type inequality for the polar of mixed complex projection bodies.

Let K denote a convex body (compact, convex subset with non-empty) in n-dimensional Euclidean space $\mathbf{R}^n$. The set of all convex bodies in $\mathbf{R}^n$ is written as $\mathcal{K}^n$. Let $\mathcal{K}_o^n$ denote the set of convex bodies containing the origin in their interiors. We write B for the unit ball centered at the origin and S^{n-1} for unit sphere in $\mathbf{R}^n$. We also use $V(K)$ to denote the n-dimensional volume of the body K.

If $K \in \mathcal{K}^n$, then its support function, $h_K = h(K, \cdot\,) : \mathbf{R}^n \to (-\infty, \infty)$, is defined by (see [16]–[17])

$$h(K, \boldsymbol{x}) = \max\{\boldsymbol{x} \cdot \boldsymbol{y} : \boldsymbol{y} \in K\}, \qquad \boldsymbol{x} \in \mathbf{R}^n,$$

where $\boldsymbol{x} \cdot \boldsymbol{y}$ denotes the standard inner product of $\boldsymbol{x}$ and $\boldsymbol{y}$. Obviously, $h(\lambda K, \cdot\,) = \lambda h(K, \cdot\,)$, where λ is a positive constant. The concept of projection body was introduced by Minkowski in the late nineteenth century to early twentieth century. For $K \in \mathcal{K}^n$, the projection body of K, $\varPi K$, is the origin-symmetric convex body whose support function is defined by (see [16])

$$h(\varPi K, \boldsymbol{u}) = \frac{1}{2} \int_{S^{n-1}} |\boldsymbol{u} \cdot \boldsymbol{v}| \mathrm{d}S(K, \boldsymbol{v}),$$

where all $\boldsymbol{u} \in S^{n-1}$ and $S(K, \cdot\,)$ is the surface area measure of K.

Mixed projection bodies were introduced in the classical volume by Bonnesen and Fenchel[18]. They are related to ordinary projection bodies in the same way that mixed volumes are related to ordinary volume.

For $K_1, \cdots, K_{n-1} \in \mathcal{K}^n$, the mixed projection body, $\varPi(K_1, \cdots, K_{n-1})$, is defined by (see [16])

$$h(\varPi(K_1, \cdots, K_{n-1}), \boldsymbol{u}) = \frac{1}{2} \int_{S^{n-1}} |\boldsymbol{u} \cdot \boldsymbol{v}| \mathrm{d}S(K_1, \cdots, K_{n-1}, \boldsymbol{v})$$

for $\boldsymbol{u} \in S^{n-1}$. Here $S(K_1, \cdots, K_{n-1}, \cdot\,)$ is the mixed surface area measure of $K_1, \cdots, K_{n-1}$.

Since the late 60s of the last century, Petty[19], Schneider[20] and Bolker[21] have renewed interest in the research of projection bodies. The research of projection bodies and mixed projection bodies have attracted many scholars' attention, a wealth of researches collected in two good books (see [16] and [17]).