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Holomorphic Curves into $\mathbb{P}^N(\mathbf{C})$ That Share a Set of Moving Hypersurfaces

YANG LIU
(*School of Mathematics & Physics Science and Engineering, Anhui University of Technology, Maanshan, Anhui, 243032*)

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Abstract: Let $\mathcal{F}$ be a family of holomorphic curves of a domain D in $\mathbf{C}$ into a closed subset X in $\mathbb{P}^N(\mathbf{C})$. Let $Q_1(z), \cdots, Q_{2t+1}(z)$ be moving hypersurfaces in $\mathbb{P}^N(\mathbf{C})$ located in pointwise t-subgeneral position with respect to X. If each pair of curves f and g in $\mathcal{F}$ share the set $\{Q_1(z), \cdots, Q_{2t+1}(z)\}$, then $\mathcal{F}$ is normal on D. This result greatly extend some earlier theorems related to Montel's criterion.

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1 Introduction and Statements

One of the most striking results in complex analysis is the Picard's little theorem which can be stated as follows (see [1]–[2]).

Theorem A *A meromorphic function in the complex plane* $\mathbf{C}$ *omits three distinct points in the extend complex plane is a constant.*

Let D be a domain in $\mathbf{C}$ and $\mathcal{F}$ a family of meromorphic functions defined in D. $\mathcal{F}$ is said to be normal in D, in the sense of Montel, if each sequence in $\mathcal{F}$ contains a subsequence, which converges spherically locally uniformly in D to a meromorphic function or ∞ (see [1]–[3]). The normality corresponding to Theorem A can be stated as follows.

Theorem B[4] *Let* $\mathcal{F}$ *be a family of meromorphic functions on a plane domain* D *which omit three distinct values* a, b, c *in the extend complex plane. Then* $\mathcal{F}$ *is normal on* D.

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E-mail address: yangliu20062006@126.com (Yang L).

Note the condition that each function f in $\mathcal{F}$ omits three values a, b, c means that the pre-image set $f^{-1}(\{a, b, c\}) = \emptyset$, Fang and Hong[5] extended Theorem B and obtained

Theorem C[5] *Let $\mathcal{F}$ be a family of meromorphic functions on a plane domain D, and let a, b, c be three distinct values in the extended complex plane. If each pair of functions f and g in $\mathcal{F}$, $f^{-1}(\{a, b, c\}) = g^{-1}(\{a, b, c\})$, then $\mathcal{F}$ is normal on D.*

In order to state the results, we recall some facts firstly.

Definition 1.1 *Let $Q_1, \cdots, Q_q$ $(q \geq t+1)$ be hypersurfaces in the complex projective space $\mathbb{P}^N(\mathbf{C})$ and $X \subseteq \mathbb{P}^N(\mathbf{C})$ be a closed set (with respect to the usual topology of a real manifold of dimension $2N$). We say that the hypersurfaces are located in t-subgeneral position with respect to X, if for any $1 \leq j_0 < \cdots < j_t \leq q$,*

$$X \cap Q_{j_0} \cap \cdots \cap Q_{j_t} = \emptyset.$$

In the particular case $X = \mathbb{P}^N(\mathbf{C})$, we have

Definition 1.2 *Let $Q_1, \cdots, Q_q$ $(q \geq t+1)$ be hypersurfaces in $\mathbb{P}^N(\mathbf{C})$. We say that the hypersurfaces are located in t-subgeneral position if for any $1 \leq j_0 < \cdots < j_t \leq q$,*

$$Q_{j_0} \cap \cdots \cap Q_{j_t} = \emptyset.$$

If a set of hypersurfaces in $\mathbb{P}^N(\mathbf{C})$ are located in N-subgeneral position, we also say that they are located in general position.

Remark 1.1[6] Neither the dimension of X nor the dimension of the ambient projective space is important in the formulation of Definition 1.1. Only the intersection pattern in the assumption of Definition 1.1 is relevant.

We do not assume $t \geq N$ in the assumption of Definition 1.1, but the assumption of Definition 1.1 can be satisfied only if $t \geq N$.

In 1972, Fujimoto[7] and Green[8] obtained the following Picard-type theorem for holomorphic mappings of several complex variables into $\mathbb{P}^N(\mathbf{C})$ related to hyperplanes.

Theorem D[7],[8] *Suppose that f is a holomorphic mapping from $\mathbf{C}^m$ into $\mathbb{P}^N(\mathbf{C})$. If f omits $2N+1$ hyperplanes in $\mathbb{P}^N(\mathbf{C})$ located in general position, then f is constant.*

In the case of hypersurfaces, Eremenko[6] obtained a Picard-type theorem for holomorphic curves into $\mathbb{P}^N(\mathbf{C})$ as follows.

Theorem E[6] *Let X be a closed subset of $\mathbb{P}^N(\mathbf{C})$ and let $Q_1, \cdots, Q_{2t+1}$ be hypersurfaces in $\mathbb{P}^N(\mathbf{C})$ located in t-subgeneral position with respect to X. Then every holomorphic mapping f from $\mathbf{C}$ into $X - \bigcup_{j=1}^{2t+1} Q_j$ is constant.*

Let Q be a fixed hypersurface of degree d in $\mathbb{P}^N(\mathbf{C})$, which is defined by a homogeneous polynomial $P(x_0, \cdots, x_N) \in \mathbf{C}[x_0, \cdots, x_N]$, i.e.,

$$Q = \{[w_0 \colon \cdots : w_N] \in \mathbb{P}^N(\mathbf{C});\ P(w_0, \cdots, w_N) = 0\}.$$