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A Nearly Analytic Discrete Method for One-dimensional Unsteady Convection-dominated Diffusion Equations

Kim Yon-chol, Yun Nam and Chai Dong-ho
(*Institute of Mathematics, Academy of Sciences of DPR Korea, Pyongyang, DPR Korea*)

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Abstract: In this paper, a nearly analytic discretization method for one-dimensional linear unsteady convection-dominated diffusion equations and viscous Burgers' equation as one of the nonlinear equation is considered. In the case of linear equations, we find the local truncation error of the scheme is $O(\tau^2 + h^4)$ and consider the stability analysis of the method on the basis of the classical von Neumann's theory. In addition, the nearly analytic discretization method for the one-dimensional viscous Burgers' equation is also constructed. The numerical experiments are performed for several benchmark problems presented in some literatures to illustrate the theoretical results. Theoretical and numerical results show that our method is to be higher accurate and nonoscillatory and might be helpful particularly in computations for the unsteady convection-dominated diffusion problems.

Key words: convection-dominated diffusion equation, nearly analytic discretization method, analysis of the stability

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1 Introduction

This paper is focused on numerical methods for convection-diffusion problems that represent mathematical models for a number of (physical) processes in fluid mechanics, astrophysics, meteorology, multiphase flow in oil reservoirs, polymer flow, financial modeling, and many other areas. Computing solutions of these problems is an important and challenging problem, especially in the convection dominated case, in which viscous layers are so thin that classical

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E-mail address: yonchol126@163.com (Kim Y C).

finite difference method or finite element method can cause severe numerical unstabilities. If an insufficient amount of physical diffusion is compensated by an excessive numerical viscosity, these methods are typically stable, but the resolution may be severely affected. At the same time, the use of dispersive schemes may cause spurious oscillations that may, in turn, trigger numerical instabilities.

A number of numerical methods for the convection-dominated diffusion equations are developed, where the leading numerical methods include stabilization method, characteristic method and discontinuous Galerkin method, adaptive mesh method (see [1]–[5]). In addition, the exponentially-fitted method is also constructed (see [6]). The main focus in these methods is to construct the method to be more accurate, high stability and uniform convergent for the diffusion parameter.

In this paper, we consider a nearly analytic discretization method (NADM) for the convection-dominated diffusion problems and viscous Burgers' equation.

The NADM was developed for the solution of the two types of the hyperbolic and the parabolic equations in 1991 by Kondoh[7]. The key idea behind the NADM is that the order of accuracy of the approximate solution of the partial equation can be increased if the numerical process is implemented by using the properties of the differential relations and solution of the equation as much as possible without direct discretization as finite difference or finite element method (see [7]–[9]).

In this paper, we construct the NADM with the local truncation error $O(\tau^2 + h^4)$ and consider the stability analysis of the NADM by using von Neumann theory for the different values of singulary perturbed parameters in the case of the singulary perturbed linear convection-diffusion equations. In addition, we construct NADM for the viscous Burgers' equation.

We can observe the method is very suitable for all cases. In order to illustrate the efficiency of the proposed method, some numerical experiments are performed for linear problems and nonlinear problems such as Burgers' equation presented in the previous literatures.

The rest of this paper is as follows.

In Section 2, we construct the NADM for the one-dimensional linear convection-dominated diffusion equation and find the local truncation error is $O(\tau^2 + h^4)$.

One of the major issues in development of numerical schemes used to solve the convection-dominated diffusion equation is stability. For the obtained explicit scheme, we make an amplification matrix $\boldsymbol{A}$ from von Neumann theory and find the relation between the space step h and time step τ for the stability.

Finally, a numerical scheme for the Burgers' equation is presented.

In Section 3, the numerical experiments for several benchmark problems in the previous literatures are implemented.

The obtained theoretical and numerical results demonstrate that the NADM has high accuracy and less numerical dispersion for the diffusion parameter and might be helpful particularly in computations for the convection-dominated diffusion equations.