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Common Fixed Points for Two Mappings with Implicit-linear Contractions on Partially Ordered 2-metric Spaces

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Abstract: In this paper, we introduce a new class $\mathcal{U}$ of 3-dimensional real functions, use $\mathcal{U}$ and a 2-dimensional real function ϕ to construct a new implicit-linear contractive condition and obtain some existence theorems of common fixed points for two mappings on partially ordered 2-metric spaces and give a sufficient condition under which there exists a unique common fixed point. The obtained results goodly generalize and improve the corresponding conclusions in references.

Key words: 2-metric space, class $\mathcal{U}$ of functions, implicit-linear contraction, common fixed point

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1 Introduction and Preliminaries

The following definition can be found in [1].

Definition 1.1 *Let $(X, \preceq, d)$ be a partially ordered 2-metric space, $S, T: X \to X$ two maps. S, T are said to be weakly C^*-contractive maps if there exists a continuous function $\varphi: [0, +\infty)^2 \to [0, +\infty)$ with $\varphi(s, t) = 0 \iff s = t = 0$ such that for any $x, y, a \in X$ with $x \preceq y$ or $y \preceq x$,*

$$d(Sx, Ty, a) \leq kd(x, y, a) + l[d(x, Ty, a) + d(y, Sx, a)] - \varphi(d(x, Ty, a), d(y, Sx, a)),$$

where k and l are two real numbers satisfying $l > 0$ and $0 < k + l \leq 1 - l$.

Piao[1] obtained the existence theorems of common fixed points for two weakly C^*-contractive mappings S and T under the continuous and non-continuous conditions and

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gave a sufficient condition under which there exists a unique common fixed point. The obtained results are not only the generalizations and improvements of the corresponding results in [2], but also the supplementary results of the corresponding results in [3]–[13].

In this paper, we introduce a class $\mathcal{U}$ of 3-dimensional real functions and define a new contractive condition by replacing the linear condition in Definition 1.1 with a implicit condition and discuss the existence problems of common fixed points for two mappings and give a sufficient condition under which there exists a unique common fixed point.

Definition 1.2[3] *A 2-metric space (X, d) consists of a nonempty set X and a function $d\colon X \times X \times X \to [0, +\infty)$ such that*

(i) *for distant elements $x, y \in X$, there exists a $u \in X$ such that $d(x, y, u) \neq 0$;*

(ii) *$d(x, y, z) = 0$ if and only if at least two elements in $\{x, y, z\}$ are equal;*

(iii) *$d(x, y, z) = d(u, v, w)$, where $\{u, v, w\}$ is any permutation of $\{x, y, z\}$;*

(iv) *$d(x, y, z) \leq d(x, y, u) + d(x, u, z) + d(u, y, z)$ for all $x, y, z, u \in X$.*

Definition 1.3[3] *A sequence $\{x_n\}_{n\in\mathbf{N}}$ in 2-metric space (X, d) is said to be a Cauchy sequence, if for each $\varepsilon > 0$ there exists a positive integer $N \in \mathbf{N}$ such that $d(x_n, x_m, a) < \varepsilon$ for all $a \in X$ and $n, m > N$. A sequence $\{x_n\}_{n\in\mathbf{N}}$ is said to be convergent to $x \in X$, if for each $a \in X$, $\lim\limits_{n\to+\infty} d(x_n, x, a) = 0$. And we write that $x_n \to x$ and call x the limit of $\{x_n\}_{n\in\mathbf{N}}$. A 2-metric space (X, d) is said to be complete, if every Cauchy sequence in X is convergent.*

Definition 1.4[6] *Let (X, d) be a 2-metric space, and $a, b \in X$, $r > 0$. The set*

$$B(a, b;\ r) = \{x \in X\colon d(a, b, x) < r\}$$

is said to be a 2-ball with centers a and b and radius r. Each 2-metric d on X generalizes a topology τ on X whose base is the family of 2-balls. τ is said to be a 2-metric topology.

Lemma 1.1[10] *If a sequence $\{x_n\}_{n\in\mathbf{N}}$ in a 2-metric space (X, d) is convergent to x, then*

$$\lim_{n\to+\infty} d(x_n, b, c) = d(x, b, c), \qquad b, c \in X.$$

Lemma 1.2[13] *Let $\{x_n\}_{n\in\mathbf{N}}$ be a sequence in (X, d) satisfying*

$$\lim_{n\to\infty} d(x_n, x_{n+1}, a) = 0, \qquad a \in X.$$

If $\{x_n\}$ is not Cauchy, then there exist $a \in X$ and $\epsilon > 0$ such that for any $i \in \mathbf{N}$, there exist $m(i), n(i) \in \mathbf{N}$ with $m(i) > n(i) > i$, $d(x_{m(i)}, x_{n(i)}, a) > \epsilon$, but $d(x_{m(i)-1}, x_{n(i)}, a) \leq \epsilon$.

Lemma 1.3[6] *$\lim\limits_{n\to\infty} x_n = x$ in 2-metric space (X, d) if and only if $\lim\limits_{n\to\infty} x_n = x$ in 2-metric topology space X.*

Lemma 1.4[6] *Let X and Y be two 2-metric spaces and $T\colon X \to Y$ a map. If T is continuous, then $\lim\limits_{n\to\infty} x_n = x$ implies $\lim\limits_{n\to\infty} Tx_n = Tx$.*

Lemma 1.5[6] *Each 2-metric space is T_2-space.*