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Exact Solutions to the Bidirectional SK-Ramani Equation

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Abstract: In this paper, the bidirectional SK-Ramani equation is investigated by means of the extended homoclinic test approach and Riemann theta function method, respectively. Based on the Hirota bilinear method, exact solutions including one-soliton wave solution are obtained by using the extended homoclinic approach and one-periodic wave solution is constructed by using the Riemann theta function method. A limiting procedure is presented to analyze in detail the relations between the one periodic wave solution and one-soliton solution.

Key words: Hirota bilinear method; bidirectional Sawada-Kotera equation; extended homoclinic test approach; Riemann theta function
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1 Introduction

In this paper, we focus on the following nonlinear evolution equation (NLEE)

$$5\partial_x^{-1}u_{tt} + 5u_{xxt} - 15uu_t - 15u_x\partial_x^{-1}u_t - 45u^2u_x + 15u_xu_{xx} + 15uu_{3x} - u_{5x} = 0, \quad (1.1)$$

which was formulated in [1] as a bidirectional counterpart of the classical Sawada-Kotera (SK) equation

$$u_t + 45u^2u_x - 15u_xu_{xx} - 15uu_{xxx} + u_{xxxxx} = 0. \quad (1.2)$$

It has been shown that the bilinear form of equation (1.1) is identical to the well-known Ramani equation (see [2]). Therefore, equation (1.1) is also called bSK-Ramani equation (see [3]). In [4], the quasi-multisoliton and bidirectional solitary wave solutions of bSK-Ramani equation (1.1) are obtained by using the Hirota's direct method (see [5]). In [6], its

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periodic solitary wave solutions are obtained by means of Hirota's direct method and ansätz function method. In [7], the Adomian decomposition method is also used to construct the soliton solutions and doubly-periodic solutions of bSK-Ramani equation (1.1).

It is well known that the Hirota's direct method (see [5]) is a powerful tool to construct exact solutions for NLEEs. Once a NLEE is written in bilinear forms, then multi-soliton solutions and rational solutions can be obtained. In [8] and [9], the Hirota's method is extended to directly construct periodic wave solutions of NLEEs with the help of Riemann theta functions. To our knowledge, there is no literature to investigate bSK-Ramani equation (1.1) by using Riemann theta functions. In this paper, based on a more general bilinear form of bSK-Ramani equation (1.1), we give some exact solutions by using the Riemann theta functions method (see [8]–[9]) and extended homoclinic test approach (see [6], [10]–[11]).

The organization of this paper is as follows. In Section 2, some exact solutions of bSK-Ramani equation (1.1) are obtained by using Hirota's direct method and extended homoclinic test approach. In Section 3, one-periodic wave solution of bSK-Ramani equation (1.1) is obtained with the help of Riemann theta function method. Moreover, a limiting procedure is applied to analyze asymptotic behaviour of the one-periodic wave solution, it is proved that the one-periodic wave solution tends to the one-soliton solution. Some conclusions are provided in Section 4.

2 Extended Homoclinic Test Approach to bSK-Ramani Equation (1.1)

In [6], Liu and Dai obtained some exact and periodic solitary wave solutions of bSK-Ramani equation (1.1) with its bilinear form expressed as

$$(5D_t^2 + 5D_x^3 D_t - D_x^6) f \cdot f = 0, \tag{2.1}$$

where the bilinear operator D is defined by

$$D_x^n D_t^m f \cdot g = (\partial_x - \partial_{x'})^n (\partial_t - \partial_{t'})^m f(x,t) g(x',t') \Big|_{x'=x, t'=t.} \tag{2.2}$$

By the following transformation

$$u = u_0 - 2\partial_x^2 \ln f, \tag{2.3}$$

where u_0 is a constant solution of bSK-Ramani equation (1.1), then equation (1.1) can be translated into the following bilinear form

$$(5D_t^2 + 5D_x^3 - D_x^6 - 45u_0 D_x^2 - 15u_0 D_x D_t + 15u_0 D_x^4 + c) f \cdot f = 0, \tag{2.4}$$

in which c is an integration constant. Assuming $c = 0$, yields

$$(5D_t^2 + 5D_x^3 - D_x^6 - 45u_0 D_x^2 - 15u_0 D_x D_t + 15u_0 D_x^4) f \cdot f = 0. \tag{2.5}$$

Based on the extended homoclinic test approach (see [6], [10] and [11]), we suppose that the solution of equation (2.5) takes the form

$$f = \mathrm{e}^{-\xi_1} + c_1 \cos(\xi_2) + c_2 \cosh(\xi_3) + c_3 \mathrm{e}^{\xi_1}, \tag{2.6}$$

where $\xi_j = \kappa_j x + \omega_j t + \delta_j$, and κ_j, ω_j, c_j $(j = 1, 2, 3)$ are constants to be determined, δ_j are free constants. Substituting the ansätz equation (2.6) into the bilinear equation (2.5) and equating all the coefficients of different powers of $\mathrm{e}^{-\xi_1}$, e^{ξ_1}, $\cos(\xi_2)$, $\sin(\xi_2)$, $\cosh(\xi_3)$,