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A Note on the Stability of K-g-frames

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Abstract: In this paper, we present a new stability theorem on the perturbation of K-g-frames by using operator theory methods. The result we obtained improves one corresponding conclusion of other authors.

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1 Introduction

A frame for a Hilbert space was discovered in 1950's by Duffin and Schaeffer[1], which has made great contributions to various fields because of its nice properties, the reader can examine [2]–[8] for background and details of frames. G-frames, proposed by Sun[9], generalize the notion of frames extensively, which possess some distinct properties though they share many similar properties with frames (see [10] and [11]).

A K-frame is an extension of a frame, which emerged in the work on atomic systems for operators due to Găvruţa[12], and the results involved show us that the properties of K-frames are quite different from those for frames owing to the linear bounded operator K (see also [13]–[16]).

The idea of Găvruţa has been applied to the case of g-frames by Xiao *et al.*[17] and thus providing us the concept of K-g-frames, which have already attracted many researchers' interest due to their potential flexibility (see [18]–[21] for example). In this paper we pay attention to the stability of K-g-frames, and the motivation derives from an observation on one stability result for K-g-frames, Theorem 4.1 in [21], recently obtained by Hua and Huang. In the proof the authors asserted that the frame operator of the involved K-g-frame is invertible on the whole space, which plays a key role in their proof to show the lower

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K-g-frame bound condition stated in the theorem. In reality, however, the invertibility of the frame operator of a K-g-frame is absent for the whole space, since a K-g-frame is not necessarily a g-frame (see Example 3.1 for details). The purpose of this paper is to provide an improvement to their result.

Throughout this paper, the notations $\mathscr{H}$ and $\mathscr{K}$ are reserved for two Hilbert spaces, and $\{\mathscr{K}_j\}_{j\in\mathbb{J}}$ is used to denote a sequence of closed subspaces of $\mathscr{K}$, where the index set $\mathbb{J}$ is finite or countable. The family of all linear bounded operators from $\mathscr{H}$ to $\mathscr{K}$ is designated as $B(\mathscr{H}, \mathscr{K})$, which is abbreviated to $B(\mathscr{H})$ if $\mathscr{H} = \mathscr{K}$. The notation $\mathscr{R}(\theta)$ designates the range of $\theta \in B(\mathscr{H}, \mathscr{K})$.

Let $\ell^2(\{\mathscr{K}_j\}_{j\in\mathbb{J}})$ be the Hilbert space defined by

$$\ell^2(\{\mathscr{K}_j\}_{j\in\mathbb{J}}) = \left\{\{g_j\}_{j\in\mathbb{J}} \colon g_j \in \mathscr{K}_j,\ j \in \mathbb{J}, \text{ and } \sum_{j\in\mathbb{J}} \|g_j\|^2 < \infty\right\},$$

where the inner product is given by

$$\langle\{f_j\}_{j\in\mathbb{J}}, \{g_j\}_{j\in\mathbb{J}}\rangle = \sum_{j\in\mathbb{J}} \langle f_j, g_j\rangle.$$

For a sequence of linear bounded operators $\{\Lambda_j\}_{j\in\mathbb{J}}$ from $\mathscr{H}$ into $\mathscr{K}_j$, let $\mathscr{H}^\Lambda$ be the set defined by

$$\mathscr{H}^\Lambda = \overline{\left\{\sum_{j\in\mathbb{I}} \Lambda_j^* g_j \text{ for any finite } \mathbb{I} \subset \mathbb{J} \text{ and } g_j \in \mathscr{K}_j,\ j \in \mathbb{I}\right\}}.$$

2 Preliminaries

In this section we mainly collect some basic definitions and properties for K-g-frames.

Definition 2.1 *Suppose $K \in B(\mathscr{H})$. One says that a family $\{\Lambda_j \in B(\mathscr{H}, \mathscr{K}_j)\}_{j\in\mathbb{J}}$ is a K-g-frame for $\mathscr{H}$ with respect to $\{\mathscr{K}_j\}_{j\in\mathbb{J}}$ if there exist $0 < C \le D < \infty$ such that*

$$C\|K^* f\|^2 \leqslant \sum_{j\in\mathbb{J}} \|\Lambda_j f\|^2 \leqslant D\|f\|^2, \qquad f \in \mathscr{H}. \tag{2.1}$$

The constants C and D are called, respectively, the lower and upper K-g-frame bounds.

Remark 2.1 If K is equal to the identity operator on $\mathscr{H}$, $\mathrm{Id}_{\mathscr{H}}$, then a K-g-frame turns to be a g-frame.

In general, if $\{\Lambda_j\}_{j\in\mathbb{J}}$ satisfies the inequality to the right in (2.1), we say that $\{\Lambda_j\}_{j\in\mathbb{J}}$ is a D-g-Bessel sequence for $\mathscr{H}$ with respect to $\{\mathscr{K}_j\}_{j\in\mathbb{J}}$, associated with which there is a linear bounded operator, called the analysis operator of $\{\Lambda_j\}_{j\in\mathbb{J}}$, defined by

$$U_\Lambda \colon \mathscr{H} \to \ell^2(\{\mathscr{K}_j\}_{j\in\mathbb{J}}), \qquad U_\Lambda f = \{\Lambda_j f\}_{j\in\mathbb{J}}. \tag{2.2}$$

The adjoint operator

$$U_\Lambda^* \colon \ell^2(\{\mathscr{K}_j\}_{j\in\mathbb{J}}) \to \mathscr{H}$$