

REVIEW ARTICLE

On Current Developments in Partial Differential Equations

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Received 11 February 2020; Accepted 27 February 2020

Abstract. The article is resulted from a round table discussion. The first part of it contains an overview on some important recent developments in the field of analysis and partial differential equations. It is a slightly expanded version of a presentation given by the author at the round table and a briefly summary of speeches by several others. The second part is a supplement of various details written by some researchers in China working on related topics.

AMS subject classifications: 35Q30

Key words: Partial differential equations, harmonic analysis, geometric measure theory.

1 An overview

During the period of January 5–15, 2020, there was a special program on the analysis and partial differential equations at the Southern University of Science and Technology of China organized by Professor Tao Tang, Xiao-Ming Wang, Linlin Su and myself. In the first seven days, there were special lecture series on the theory of quantitative periodic and stochastic homogenizations and its applications. The two principle lecturers are Professor T. Kuusi and Professor Zhongwei Shen. It is followed by a conference “On the recent trend in the analysis of partial differential equations”. The program also included a special Round Table discussion on some important current developments in the field of analysis and partial

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differential equations; and to have a discussion on the research and education programs for this field in Chinese universities.

I am very happy that we had such a round table discussions. Many friends and colleagues working in the field got together having wonderful conversations and sharing stories about old and new events. Naturally, many recent research works in field were also brought into our attention. At such a round table, each of us can open his/her mind to express freely their individual and particular views on the past and current developments and to provide thoughts for the future if that would be possible. Thinking back, it's truly worth to have such an occasion, once a while. Last year was also special in some sense at my personal level. I arrived at 60, and it's a natural time to have a reflection. I left China in 1981 for a graduate study in the USA and I have since stayed and worked here. My first returning to China after my Ph.D was in 1989, so it is also the 30th anniversary for me to be participated in and sometime to co-organize some of academic activities in China during my academic breaks or sabbatical leaves. There are a lot of great memories and a lot of wonderful old and young friends over all these years. Many have also become scientific collaborators. It has become an important part of life and it may be why I (and may be almost all of us) love mathematics.

At the round table discussions, many of us have noticed that our generations have probably not done that well as one would have hoped; and as we shall believe, history may not necessary be so friendly. For our teacher's generation 40 years ago, China is at a completely different state. Excellent education and research programs are always difficult to build, and particularly so during the school years of our generation. For now, we no longer have such excuses for not catching up forefronts of research and not to do high quality and more fundamental researches. We have also no excuses not to do high quality teachings for next generations. One can take a look at a simple statistics in China: there are thousands of researchers in the field of Analysis & PDE, and in each year (over the past decades) there have been also in thousands of research papers got published. On the other hand, it is evident over the years that the impacts and visibilities of these works are relatively limited. It is not about the lack of communications or supports in academic activities. In fact, it is not something that one can say that it is normal. One should do something about it strategically. At least, one needs to know why? What and how one can do about it?

Looking outside the field of analysis and PDEs, the field of computational and applied math seems doing much better in China via comparisons. It may be also for several other subjects such as differential geometry /geometric analysis. In fact, a very noticeable advances and sharp improvements have been made in certain parts of pure and abstract mathematics. So we could ask: does the field