Commun. Math. Res.
doi: 10.4208/cmr.2020-0003

Vol. **36**, No. 1, pp. 51-67
March 2020

Equivalence Relation between Initial Values and Solutions for Evolution p-Laplacian Equation in Unbounded Space

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Received 12 February 2020; Accepted 1 March 2020

Abstract. In this paper, an equivalence relation between the ω-limit set of initial values and the ω-limit set of solutions is established for the Cauchy problem of evolution p-Laplacian equation in the unbounded space $Y_\sigma(\mathbb{R}^N)$. To overcome the difficulties caused by the nonlinearity of the equation and the unbounded solutions, we establish the propagation estimate and the growth estimate for the solutions. It will be demonstrated that the equivalence relation can be used to study the asymptotic behavior of solutions.

AMS subject classifications: 35K55, 35B40

Key words: Asymptotic behavior, evolution p-Laplacian equation, unbounded function, propagation estimate, growth estimate.

1 Introduction

In this paper, we consider the asymptotic behavior of solutions for the Cauchy problem of the evolution p-Laplacian equation

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$$\frac{\partial u}{\partial t} - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = 0 \quad \text{in } \mathbb{R}^N \times (0,\infty), \tag{1.1}$$
$$u(x,0) = u_0(x) \quad \text{in } \mathbb{R}^N, \tag{1.2}$$

where $p>2$ and the nonnegative initial value

$$u_0 \in Y_\sigma(\mathbb{R}^N) \equiv \left\{ \varphi \in C(\mathbb{R}^N) : \lim_{|x|\to\infty} (1+|x|^2)^{-\frac{\sigma}{2}} \varphi(x) = 0 \right\}$$

with $0 \le \sigma < \frac{p}{p-2}$.

Since the beginning of this century, there has been a great interest in the complicate asymptotic behavior of solutions for some evolution equations [1–8]. To do this, a successful method is to establish the relation between the initial values and the solutions for the evolution equations in some Banach spaces. In 2002, it was Vázquez and Zuazua [9] who first considered the relation between the ω-limit set of initial values and the ω-limit set of solutions to the problem (1.1)-(1.2) in the bounded space $L^\infty(\mathbb{R}^N)$. They found that the set of accumulation points of the rescaled solutions $u(t^{\frac{1}{p}}x,t)$ to the problem (1.1)-(1.2) in $L^\infty_{\text{loc}}(\mathbb{R}^N)$ as $t\to\infty$ coincides with the set of $\{S(1)(\varphi)\}$, where φ ranges over the set of the accumulation points as $\lambda\to\infty$ of the family $\{u_0(\lambda x);\ \lambda>0\}$ in the weak-star topology of $L^\infty(\mathbb{R}^N)$. By using this relation, they proved that the complicated asymptotic behavior can happen in the solutions. Later Cazenave, Dickstein and Weissler [10–13] investigated the relation between the rescaled solutions $t^{\frac{\mu}{2}}u(t^\beta x,t)$ $(\mu,\beta>0)$ and the initial values for the heat equation in bounded space $C_0(\mathbb{R}^N)$. They also used these relation to investigate the complicated asymptotic behavior of solutions. They also study the complicated asymptotic behavior of solutions for the Navier-Stokes equations and the Schrödinger equation [14, 15]. In our recent papers [16,17], we revealed that there exists an equivalence relation between the ω-limit set of initial values and the ω-limit set of rescaled solutions $t^{\frac{\mu}{2}}u(t^\beta x,t)$ $(\mu,\beta>0)$ in bounded space $C_0(\mathbb{R}^N)$, and use this relation to study the complicate asymptotic behavior of solutions for the Cauchy problem of the porous medium equation and the Cauchy problem of the evolution p-Laplacian equation respectively. The studies of other asymptotic behavior of solutions for the evolution equations can be found in [18–23].

Note that the relations in the above works are only considered in some bounded spaces. It follows from the existence theory for the evolution p-Laplacian equation that the solutions of the problem (1.1)-(1.2) are global even if the initial data belong to some unbounded spaces [24–26]. Our interest here is to