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Stochastic Second-Order Two-Scale Method for Predicting the Mechanical Properties of Composite Materials with Random Interpenetrating Phase

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Abstract. In this paper, a stochastic second-order two-scale (SSOTS) method is proposed for predicting the non-deterministic mechanical properties of composites with random interpenetrating phase. Firstly, based on random morphology description functions (RMDF), the randomness of the material properties of the constituents as well as the correlation among these random properties are fully characterized through the topologies of the constituents. Then, by virtue of multiscale asymptotic analysis, the random effective quantities such as stiffness parameters and strength parameters along with their numerical computation formulae are derived by a SSOTS strategy combined with the Monte-Carlo method. Finally, the SSOTS method developed in this paper shows an excellent computational accuracy, and therefore present an important advance towards computationally efficient multiscale modeling frameworks considering microstructure uncertainties.

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1 Introduction

With the rapid development of modern material science and technology, it is of great practical value and theoretical significance to understand the response of composite materials possessing highly random interpenetrating phase since they are widely encountered in engineering applications [1, 2]. Due to the random nature of the material's microstructure, the improvement of mechanical performance of composites requires considerations that go far beyond the selection of the overall composition of the composites. They demand a rigorous treatment of their complex hierarchical uncertainty microstructure [3]. As we known, multiple phases and their spatial distribution in the material microstructure are expected to play a dominant role in controlling the non-deterministic effective mechanical properties and performance. Therefore, there is a critical need for the development and verification of high accuracy multiscale modeling strategies to obtain certain desired mechanical properties by the design of a microstructure.

The multiscale method for composites with periodic microstructures has been developed for several years and used widely in engineering. Among these multiscale or homogenization methods, the asymptotic expansion homogenization method has several attractive features due to its systematic and rigorous mathematical theory [4–6]. The field variables such as displacements are assumed to vary in multiple scales, and thus they are represented by asymptotic expansions in each spatial scales. It is important to emphasize that the asymptotic expansion homogenization method enables coupling of the local and global-levels solution of the problem, which converges to the solution of the homogenized problem as the period of unit cell goes to zero. Also, the local fluctuations of field variables can be easily computed from the homogenized solution and the estimation of the local errors due to homogenization is rather straightforward [7]. Moreover, researchers have applied the method for both linear and non-linear structural problems [8–11] implementing in many computational techniques such as the finite element method (FEM). This implies that the treatment of highly complex microscale composite geometries becomes feasible. However, existing methods that treat the multiscale problem for more sophisticated constitutive models employ the first-order approximation methods that are inapplicable for complex composite geometries. Furthermore, many of the computational and numerical details, such as local fluctuations, in practical and engineering applications are not able