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Periodic Solutions for a Damped Rayleigh Beam Model with Time Delay

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Abstract. Vibrations of a beam can be described as an Euler-Bernoulli beam, or as a Rayleigh beam or as a Timoshenko beam. In this paper, we establish the existence of periodic solutions in time for a damped Rayleigh beam model with time delay, which is treated as a bifurcation parameter. The main proof is based on a Lyapunov-Schmidt reduction together with the classical implicit function theorem. Moreover, we give a sufficient condition for a direction of bifurcation.

AMS subject classifications: 35L75, 37K50

Key words: Beam equations, damping, time delay, periodic solutions.

1 Introduction

This article concerns the following Rayleigh beam equation subject to damping and time delay

$$\begin{aligned} &u_{tt}(t,x)-u_{ttxx}(t,x)+u_{xxxx}(t,x)+u_t(t,x)+u(t,x)+u(t-\tau,x)\\ &=f(u(t-\tau,x),x), \quad x\in\mathbb{T}, \end{aligned} \tag{1.1}$$

where the positive constant τ stands for time delay, $\mathbb{T}:=\mathbb{R}/2\pi\mathbb{Z}$ denotes the standard 1-D real torus, and the nonlinearity $f:\mathbb{R}\times\mathbb{T}\longrightarrow\mathbb{R}$ is taken to be C^∞

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smooth. We call u_t external or viscous damping introduced by external, linear dampers.

Many structures, such as bridges, runways, rails, roadways, pipelines, etc., can be modelled as beam structures. In vibration analysis of a beam, it can be specifically described as an Euler-Bernoulli beam, or as a Rayleigh beam, or as a Timoshenko beam and so on, see [18, p.125-128]. The Rayleigh beam model is a fairly modest modification of the Euler-Bernoulli beam model, dating back some hundred years or so. To obtain it, the beam element of width δ centered at x is additionally endowed with mass moment of inertia $\delta I^\rho(x)$. Then we add $I^\rho(\partial_{tx}u(t,x))^2$ to the following energy

$$\mathcal{E}(t)=\int_0^\pi \rho(\partial_t u(t,x))^2+EI(\partial_{xx}u(t,x))^2dx$$

for the mass density ρ, Young's modulus of elasticity E, and the second moment of area of the beam's cross section I, and derive

$$\rho u_{tt}-(I^\rho u_{ttx})_x+(EIu_{xx})_{xx}=0,\quad x\in(0,\pi).$$

In real process, damping effect always exists in each dynamic system. Recently, for ϵ small enough, Kogelbauer and Haller [7] obtained spectral submanifolds (SSMs) of the Rayleigh beam equation with damping

$$u_{tt}-\mu u_{xxtt}+\alpha u_{xxxx}+\beta u_t-\gamma u_{xxt}+mu=f(u)+\epsilon g,\quad x\in(0,\pi)$$

with respect to initial data

$$u(0,x)=u_0(x),\quad u_t(0,x)=v_0(x),$$

where u_{xxt} is the so-called structural damping. For further references on the Rayleigh beam model, we can refer the readers to the articles [3,4,11,16,17,20,21,23].

Time delays arise in many applications depending not only on the present state but also on some past occurrences. Generally speaking, one must measure the state of some system at time t in order to control its motion, and then according to these measurements, apply the feedback. However the latter cannot happen instantaneously. Hence there will exist a positive time delay. Now let us state the background corresponding to partial differential equations (PDEs) subject to damping and time delay. On the one hand, the presence of delay may be a source of instability, and hence it affects the existence of attractors. Nicaise and Pignotti [12] studied the following wave equation

$$u_{tt}(t,x)-\Delta u(t,x)+\alpha u_t(t,x)+\beta u_t(t-\tau,x)=0,\quad x\in\Omega\subset\mathbb{R}^d.$$