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Global Well-Posedness of the Inviscid Heat-Conductive Resistive Compressible MHD in a Strip Domain

Yanjin Wang[1] and Zhouping Xin[2,*]

[1] *School of Mathematical Sciences, Xiamen University, Xiamen, Fujian 361005, China.*
[2] *The Institute of Mathematical Sciences, The Chinese University of Hong Kong, Shatin, NT, Hong Kong, SAR, China.*

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Abstract. This paper concerns the inviscid, heat-conductive and resistive compressible MHD system in a horizontally periodic flat strip domain. The global well-posedness of the problem around an equilibrium with the positive constant density and temperature and a uniform non-horizontal magnetic field is established, and the solution decays to the equilibrium almost exponentially. Our result reveals the strong stabilizing effect of the transversal magnetic field and resistivity as the global well-posedness of compressible inviscid heat-conductive flows in multi-D is unknown.

AMS subject classifications: 35A01, 35Q35, 76N10, 76W05

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1 Introduction

When the viscosity is neglected whereas the heat conduction and magnetic diffusion are taken into account, the dynamics of compressible electrically conducting

*Corresponding author. *Email addresses:* yanjin_wang@xmu.edu.cn (Y. J. Wang), zpxin@ims.cuhk.edu.hk (Z. P. Xin)

fluids interacting with magnetic fields can be described by the following magnetohydrodynamic system (MHD) [7,11]:

$$\begin{cases} \partial_t\rho+\operatorname{div}(\rho u)=0, \\ \partial_t(\rho u)+\operatorname{div}(\rho u\otimes u)+\nabla p=\operatorname{curl}B\times B, \\ \partial_t(\rho e)+\operatorname{div}(\rho ue)+p\operatorname{div}u-\mu\Delta\theta=\kappa|\operatorname{curl}B|^2, \\ \partial_t B=\operatorname{curl}E, \quad E=u\times B-\kappa\operatorname{curl}B, \\ \operatorname{div}B=0, \end{cases} \tag{1.1}$$

where ρ, u, θ and B denote the density, velocity, temperature and magnetic field, respectively, and E is the electric field. The fluid is assumed to obey the ideal polytropic law, so the pressure $p=R\rho\theta$ and the internal energy $e=c_v\theta$ with constants $R,c_v>0$. $\mu>0$ is the heat conduction coefficient and $\kappa>0$ is the magnetic diffusion coefficient.

The main difficulty of studying the global well-posedness of (1.1) lies in the absence of the viscosity. Similar to the Navier-Stokes equations, the viscous and resistive (incompressible and compressible) MHD system has a unique global classical solution, at least for the small initial data, see [5,9,18] for instance. On the other hand, it is remarkable that the ideal incompressible homogeneous MHD system in the whole space also admits a unique global classical solution around a nonzero uniform magnetic field [2,3,8,22]. It is then natural to ask whether the MHD systems with only the viscosity or resistivity admit global classical solutions or develop singularities in finite time. The global existence of classical solutions to the viscous and non-resistive MHD systems has been established around a nonzero uniform magnetic field. For the Cauchy problem, we refer to [1,13,15,25,27] for the incompressible homogeneous case and [24] for the 2D compressible isentropic case. For the initial boundary value problem, the global well-posedness has been proved only for the case of a horizontally flat strip domain, see [16] for the 2D incompressible homogeneous system around a uniform horizontal magnetic field and [20] for the 3D (incompressible and compressible) systems around a uniform non-horizontal magnetic field. The inviscid and resistive incompressible homogeneous 2D MHD system has a global weak solution in H^1, but the question whether such weak solutions are unique or can be improved to be global classical solutions remains open [4,10,12]. For a 2D periodic domain, [28] showed the global existence of classical solutions around a nonzero uniform magnetic field when the initial data has certain symmetries, and [23] proved a global well-posedness around the zero magnetic field.

In this paper, we consider the compressible MHD system (1.1) in the strip domain $\Omega=\mathbb{T}^2\times(0,1)$ for $\mathbb{T}=\mathbb{R}/\mathbb{Z}$, with the following boundary conditions: