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Armendariz Property of $k[x,y]$ Modulo Monomial Ideals

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Abstract. In this paper, we give equivalent conditions for the factor rings of the polynomial ring $k[x,y]$ modulo monomial ideals to be Armendariz rings, where k is a field. For an ideal I with 2 or 3 monomial generators, or n homogeneous monomial generators, such that $k[x,y]/I$ is an Armendariz ring, we characterize the minimal generator set $G(I)$ of I.

AMS subject classifications: 16S36, 16D25

Key words: Armendariz ring, polynomial ring, monomial ideal.

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1 Introduction

A ring without nonzero nilpotent elements is called reduced. Armendariz [4] noted that a reduced ring R has the following property: for each pair of polynomials $f(x)=a_0+a_1x+\cdots+a_mx^m$ and $g(x)=b_0+b_1x+\cdots+b_nx^n$ over R, $f(x)g(x)=0$ implies $a_ib_j=0$ for each i, j. Motivated by this fact, Rege and Chhawchharia [16] called a ring satisfying the property above Armendariz ring and initiated the research of Armendariz rings. Rings with Armendariz properties have been studied extensively, such as the trivial extension, the classical quotient ring, polynomial ring, skew polynomial ring, group ring and so on. For part of the related work see [1–3, 7, 9–12, 16, 18, 19]. Armendariz-like properties, such as central Armendariz, weak Armendariz, skew Armendariz and so on are also studied, for

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example [5, 6, 13–15, 17]. However, much less is known about the structure of Armendariz ring.

From the definition it is easy to see that subrings of an Armendariz ring are also Armendariz rings, but not all factor rings of an Armendariz ring are Armendariz rings. A natural question is when factor rings of an Armendariz ring are Armendariz rings. Rege and Chhawchharia [16] showed that the factor rings of a PID are also Armendariz rings. Guo *et al.* [7] proved that a UFD factoring a principle ideal gives an Armendariz ring. Anderson and Camillo [1] proved that the polynomial ring $R[x]$ over an Armendariz ring R is an Armendariz ring, and $R[x]/(x^n)$ $(n>1)$ is an Armendariz ring if and only if R is reduced, where (x^n) denotes the ideal generated by x^n; for a commutative ring R, every factor ring of R is Armendariz if and only if R is a Gauss ring. Motivated by these work on the Armendariz property of factor rings, in this paper, we investigate the Armendariz property of the polynomial ring $k[x,y]$ over a field k modulo its monomial ideal I. We give a necessary condition for the factor ring $k[x,y]/I$ to be an Armendariz ring. For I with 2, 3 monomial generators, or n homogeneous monomial generators, we give equivalent conditions for $k[x,y]/I$ to be an Armendariz ring by characterizing the minimal generator set of I.

2 Preliminaries

Throughout this paper, let k be a field and $k[x_1,\dots,x_n]$ be the polynomial ring in n variables over k. An ideal I of $k[x_1,\dots,x_n]$ is called a monomial ideal if it is generated by monomials. By [8, Proposition 1.1.6], each monomial ideal I has a unique minimal monomial set of generators, denoted by $G(I)$.

For a polynomial $f(T)\in k[x,y][T]$, we write

$$\begin{aligned} f(T) &= f_0+f_1T+\cdots+f_nT^n \\ &= x^p(u_0+u_1x+\cdots+u_mx^m), \end{aligned}$$

where $f_i\in k[x,y]$, $u_s\in k[T][y]$. For $u_s\neq 0$, denote the lowest degree of y in u_s by $l^y_{u_s}$. As an element of $k[T][x,y]$, $f(T)$ is also written as

$$f(T)=F(x,y)=F_0+F_1+\cdots+F_m,$$

where

$$F_t=\sum_{i+j=t}F_{ij}x^iy^j,\quad F_{ij}\in k[T],\quad t=0,\dots,m.$$

For an integer $n\in\mathbb{Z}$, let $\lceil n\rfloor$ denote the minimal integer which is greater than or equal to n.