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Dynamics of Poisson-Nernst-Planck Systems and Ionic Flows Through Ion Channels: A Review

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Abstract. Poisson-Nernst-Planck systems are basic models for electrodiffusion process, particularly, for ionic flows through ion channels embedded in cell membranes. In this article, we present a brief review on a geometric singular perturbation framework for analyzing the steady-state of a quasi-one-dimensional Poisson-Nernst-Planck model. The framework is based on the general geometric singular perturbed theory from nonlinear dynamical system theory and, most crucially, on the reveal of two specific structures of Poisson-Nernst-Planck systems. As a result of the geometric framework, one obtains a governing system – an algebraic system of equations that involves all physical quantities such as protein structures of membrane channels as well as boundary conditions, and hence, provides a complete platform for studying the interplay between protein structure and boundary conditions and effects on ionic flow properties. As an illustration, we will present concrete applications of the theory to several topics of biologically significant based on collaboration works with many excellent researchers.

AMS subject classifications: 34A26, 34B16, 78A35, 92C35

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1 Introduction: Ion channels, ionic flows, and models

1.1 A brief background of ion channels and ionic flows

There are excellent references on ion channels (see, e.g., [37,78]). We only provide a brief background of ion channels and ionic flows through ion channels.

What ion channels are and what they are for. Cells are basic units for all living organism. Cell membrane protects cell identity. Ion channels – "pores" of large proteins embedded in cell membranes – provide a major channel for cells to communicate with each other and the outside world to transform signals and to conduct group tasks (see, e.g., [10, 11, 21, 27, 37–45]). To some degree, an ion channel connects two large baths with ionic solutions in each with different properties. Since aqueous solutions on the two sides of the membrane are highly ionic, once a channel opens, the two baths will have essentially the same electrical potential difference as the membrane potential difference. Ions will be driven essentially by the transmembrane potential difference as well as the ionic concentration difference; that is, it is the interplay between electrical potential gradient and ion concentration gradients that causes the overall ionic flow and, in turn, creates the membrane potentials that are the carrier of nerve signals (see, e.g., [13, 27, 31, 32, 51]). It is thus critical to understand how signals are controlled by the various physical parameters.

Channel structures and properties of ionic flows. The study of ion channels consists of two related major topics: structures of ion channels and ionic flow properties. The key structure of an ion channel is the channel shape and the permanent charge in the neck of the channel. The shape of a typical ion channel could be approximated but not exactly as cylindrical-like domain with variable cross-section areas along its longitudinal axis. Within an ion channel, amino acid side chains are distributed, with acidic side chains contributing negative charges and basic side chains contributing positive charges. It is the specific of side chain distributions in an ion channel that is referred to as the permanent charge of the ion channel.

The atomic structure determines the class of mechanisms of channel functions (see, e.g., [8, 9, 12, 20, 29, 37, 75, 77]), in an important but qualitative sense, and provides the basis for describing correlations in continuum models and helps improve these continuum models to be more accurate and more realistic in significant ways. The structures being discussed are determined from crystals, with somewhat different locations of atoms and forces (or they would not crystallize), in ionic mixtures often remote from the physiological solutions in which the chan-