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The AOR-Base Splitting Modified Fixed Point Iteration for Solving Absolute Value Equations

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Abstract. Recently, Yu *et al.* presented a modified fixed point iterative (MFPI) method for solving large sparse absolute value equation (AVE). In this paper, we consider using accelerated overrelaxation (AOR) splitting to develop the modified fixed point iteration (denoted by MFPI-JS and MFPI-GSS) methods for solving AVE. Furthermore, the convergence analysis of the MFPI-JS and MFPI-GSS methods for AVE are also studied under suitable restrictions on the iteration parameters, and the functional equation between the parameter τ and matrix Q. Finally, numerical examples show that the MFPI-JS and MFPI-GSS are efficient iteration methods.

AMS subject classifications: 65H10, 65F10

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1 Introduction

Consider the solution of the system of absolute value equation (AVE)

$$Ax-|x|=b, \tag{1.1}$$

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where $A\in\mathbb{R}^{n\times n}$, $b\in\mathbb{R}^n$, and $|x|$ denotes the vector with absolute values of each component of an unknown $x\in\mathbb{R}^n$. The system (1.1) is a special case of the generalized absolute value equations of the following form:

$$Ax-B|x|=b, \tag{1.2}$$

where $B\in\mathbb{R}^{n\times n}$, was introduced by Rohn [18], and further studied in [10,14,25]. GAGE has appeared in various scientific and engineering fields, and has been widely used, such as in the analysis of linear programming problems, convex quadratic programming, linear complementarity problems and many other applications, see [17,24] and references.

Proposition 1.1 ([12,15]). *Assume that $A\in\mathbb{R}^{n\times n}$ is invertible. If $\|A^{-1}\|<1$, then the AVE* (1.1) *has a unique solution x^* for any $b\in\mathbb{R}^n$.*

The problem of finding the unique solution of the AVE (1.1) with $\|A^{-1}\|<1$ has attracted wide attention, see [2,19]. Recently, Ke *et al.* reformulated AVE (1.1) as a two-by-two block nonlinear equation and the SOR-like iterative method for solving AVE (1.1) is proposed [6,9]. In recent years, the SOR-like iterative method has been widely concerned by many researchers in the fields, and achieved significant progress. Using similar techniques, other SOR-based methods for solving AVE (1.1) are proposed [4,7,28]. To further improve the computational efficiency, Ke developed an efficient fixed-point iteration method [8], and Yu *et al.* proposed a modified fixed point iteration method [26] to solve AVE (1.1). Several other algorithms have been designed to solve the system of AVE (1.1), see [3,5,10,13,16,20,22,23,27,29–31] and references therein. For example, in [26], modified fixed point iteration method for solving AVE (1.1).

Algorithm 1 MFPI Method for the AVE (1.1).

1: Let $A\in\mathbb{R}^{n\times n}$ be a nonsingular matrix and $b\in\mathbb{R}^n$.
2: Given initial vectors $x^{(0)}\in\mathbb{R}^n$, $y^{(0)}\in\mathbb{R}^n$ and a nonsingular matrix $Q\in\mathbb{R}^{n\times n}$
3: **for** $k=0,1,2,\ldots$ **do**
4: **while** the iteration sequence $\{x^{(k)},y^{(k)}\}_{k=0}^{\infty}$ is convergent **do**
5: Compute

$$\begin{cases} x^{(k+1)}=A^{-1}(Qy^{(k)}+b), \\ y^{(k+1)}=(1-\tau)y^{(k)}+\tau Q^{-1}|x^{(k+1)}|, \end{cases} \tag{1.3}$$

where the iteration parameter $\tau>0$.
6: **end while**
7: **end for**