Communications
in
Mathematical
Research
**25**(1)(2009), 69–78

# On Gliding Hump Properties of Matrix Domains*

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Communicated by Ji You-qing

**Abstract:** In this note, we establish several results concerning the gliding hump properties of matrix domains. In order to discuss $F$-WGHP, we introduce the $UAK$-property and find that this sort of property has close relationship with $F$-WGHP. In the course of discussing $F$-WGHP and WGHP of $(c_0)_{C_n}$, we discuss the $F$-WGHP and WGHP of the almost-null sequence space $f_0$.

**Key words:** matrix domain, UAK-property, gliding hump property, sequence space
**2000 MR subject classification:** 40C05, 46A04, 46A45
**Document code:** A
**Article ID:** 1674-5647(2009)01-0069-10

## 1 Introduction

Recently, Boos and his collaborators have presented classes of infinite matrices $A$ such that the matrix domain $E_A$ has a certain gliding hump property whenever a given sequence space $E$ has this property in [1]. In this note we discuss the $F$-WGHP and WGHP of $(c_0)_{C_n}$, then give the $F$-WGHP and WGHP of the almost-null sequence space $f_0$.

The gliding hump technique of proof was originally introduced by Lebesgue (see [2]). Now this kind of method has been used to treat numerous topics in analysis, and this kind of property was generalized extensively and used to establish some important results, and you can refer to [3], [4] for detailed information. While there are known examples of sequence spaces possessing the various gliding hump properties, there are few known examples of spaces with signed gliding hump and signed $F$-gliding hump properties so it would be of interest to have constructions which provide examples of sequence spaces with

***Received date:** March 14, 2008.

various gliding hump properties. In [1], Boos and his collaborators introduced a general procedure for constructing a sequence space from a given sequence and an infinite matrix. In this note we continue to use this procedure given by J. Boos to construct examples of sequence spaces with singed $F$-weak gliding hump and $F$-weak gliding hump properties.

## 2 Notations and Preliminaries

We begin by fixing the notations and describing the general procedure which we employ for generating sequence spaces from infinite matrices. Let $E$ be a vector space of scalar sequences which contains the subspace $c_{00}$ of all sequences which are eventually 0. Let

$$A = [a_{ij}]$$

be an infinite matrix. If $x = (x_j)$ is a scalar sequence, let

$$Ax = \Big(\sum_{j=1}^{\infty} a_{ij}x_j\Big)$$

be the image of $x$ under the matrix $A$ provided each series $\sum\limits_{j=1}^{\infty} a_{ij}x_j$ converge for every $i$. We use the sequence space $E$ and matrix $A$ to generate a further sequence space. We define $E_A$ to be the vector space of all sequences $x$ such that $Ax \in E$. Then $A$ is a linear map from $E_A$ into $E$. Some of the familiar sequence spaces can be generated by this construction. In particular, $c_A$ and $(c_0)_A$ are the spaces of all sequences which are $A$-summable and $A$-summable to 0, respectively. Note, $E_A$ is an $FK$-space whenever $E$ is.

**Example 2.1** Let $B = [b_{ij}]$ be the matrix with $b_{ij} = 1$ for $j \leq i$ and $b_{ij} = 0$ otherwise. Then $l^\infty_B = bs$, the space of bounded series, and $c_B = cs$, the space of convergent series.

**Example 2.2** Let $n$ be an arbitrary nonnegative integer and $C_n = (c_{ij})$ be the matrix with $c_{ij} = 1/i$ for $n + 1 \leq j \leq i + n$ and $c_{ij} = 0$ otherwise. Then $C_n$ becomes the Cesàro matrix when $n = 0$. So we call $C_n$ to be generalized Cesàro matrix. In particular, $l^\infty_{C_n}$ is the vector space of sequences with bounded averages

$$l^\infty_{C_n} = \Big\{x : \sup_{k\in N}\Big|\frac{1}{k}\sum_{j=n}^{k+n} x_j\Big| < \infty\Big\}.$$

**Example 2.3** More generally, we consider Riesz matrices (means) $R_p$ (instead $C_0$) also known as weighed means: we consider exclusively real sequences $p = (p_k)$ with

$$p_1 > 0, \qquad p_k \geq 0 \ \ (k \in N), \qquad \text{and} \qquad P_n := \sum_{k=1}^{n} p_k \ \ (k \in N) \tag{2.1}$$

Then the Riesz matrix $R_p = (r_{ij})$ (associated with $p$) is defined by

$$r_{ij} = \begin{cases} p_j/P_i, & \text{if } j \leq i; \\ 0, & \text{otherwise.} \end{cases}$$

Note that if $p = (1, 1, \cdots)$, then $R_p = C_1$. Each Riesz matrix $R_p$ is conservative and is either regular (being equivalent to $p \notin l^1$) or coercive (see [5], Section 3.2).

**Example 2.4** Let $B_1 = [b_{ij}]$ be the matrix with $b_{ii} = 1$, $b_{i+1,i} = -1$ and $b_{ij=0}$ otherwise. Then

$$l^1_{B_1} = bv,$$

the space of sequences of bounded variation. Also $cs_{B_1} = c$ and $bs_{B_1} = l^\infty$.

**Example 2.5** Let $D_n = [d_{ij}]$ be the matrix with $d_{i,i+n} = i$, $d_{i+1,i+n} = -i$ and $d_{ij=0}$ otherwise, in which $n$ is a nonnegative integer. Then $(l^\infty_{C_0})_{D_0} = l^\infty$. Also $((c_0)_{C_0})_{D_0} = c_0$.

# 3 An Algebra Problem

In [1], the authors posed the following general (algebraic) problem:

($\mathcal{A}$) If the sequence space $E$ has the (algebraic) property P, for what matrices $A$ does the space $E_A$ have the property P?

We now consider problem ($\mathcal{A}$) for the weak gliding hump, $F$-weak gliding hump, signed weak gliding hump and signed $F$-weak gliding hump properties. In [1], the authors have discussed the problems ($\mathcal{A}$) for the weak gliding hump property and signed weak gliding hump property. They obtained some beautiful results and, in the same time, posed some open problems on these two kinds of gliding hump property; it is suggested to see [1] for more information. An interval in $N$ is a set of the form

$$[m, n] = \{k \in N : m \le k \le n\}, \qquad m \le n.$$

A sequence of intervals $(I_k)$ is increasing if for all $k$

$$\max I_k < \min I_{k+1}.$$

If $x = (x_k)$ is a scalar sequence and $I$ is an interval, then $\chi_I x$ denotes the coordinatewise product of $x$ and $\chi_I$ the characteristic function of $I$. Let $F$ be a finite subset of $N$. Then $F$ is a net if $F$ is directed by inclusion.

**Definition 3.1** *The space $E$ is said to have the signed weak gliding hump property (simply signed WGHP) if whenever $x \in E$ and $(I_k)$ is an increasing sequence of intervals, there is a strictly increasing sequence $(n_k)$ and a sequence of signs $(s_k)$, $(s_k) = \pm 1$ for every $k$, such that the coordinatewise sum*

$$\tilde{x} = \sum_{j=1}^{\infty} s_k \chi I_{n_k} x$$

*belongs to $E$. The space $E$ has the weak gliding hump property (WGHP) if the sign $s_k$ above is equal to 1 for each $k$.*

**Definition 3.2**[6] *$E$ is said to have the signed $F$-weak gliding hump property (signed $F$-WGHP) if whenever $x \in E$ and $\{\sigma_k\}$ is an increasing sequence of subsets of $F$, there is a sequence of signs $\{s_k\}$ and a subsequence $\{n_k\}$ such that the coordinatewise sum of the series $\sum\limits_{k=1}^{\infty} s_k \chi_{\sigma_{n_k}} x$ belongs to $E$; if all the signs $s_k$ are equal to 1, $E$ is said to have the $F$-WGHP.*

Of course, the difference between the signed WGHP and the property defined above is the use of increasing sequences of arbitrary finite subsets of $N$ or intervals.

**Remark 3.1** Any monotone space obviously has signed $F$-WGHP. An example which is a non-monotone space with $F$-WGHP was given in [6]. Obviously, signed $F$-WGHP ($F$-WGHP) implies signed WGHP (WGHP). The converse implication does not hold. In fact, $bs$ has signed WGHP but it does not have signed $F$-WGHP. The proof is contained in the example 31 in [6].

**Example 3.1** The space $cs$ has WGHP, but we show that it does not have $F$-WGHP. Let

$$x = \{1,\ -1,\ \frac{1}{2},\ -\frac{1}{2},\ \cdots\} \in cs.$$

Set

$$\sigma_0 = \{1\}, \quad \sigma_1 = \{3,\ 5,\ 7\}, \quad \cdots$$

where $\sigma_k$ consists of $3^k$ consecutive odd integers. Let

$$y_k = \sum_{j\in\sigma_k} x_j.$$

We show that

$$\sum_{j=0}^{\infty} y_j = \lim_n \sum_{j=0}^{n} y_j = \infty,$$

which implies that

$$\Sigma\chi_{\sigma_{n_k}} x \notin cs.$$

In fact, for each $j \in N$, we have

$$y_j = \sum_{j\in\sigma_k} x_j = \sum_{j=(3^k+1)/2}^{(3^{k+1}-1)/2} \frac{1}{j} > 3^k \frac{2}{3^{k+1}} = \frac{2}{3}.$$

A similar argument shows that $\sum \chi_{\sigma_k} x \notin cs$ for any subsequence $\{n_k\}$, so $cs$ does not have $F$-WGHP.

**Theorem 3.1** *If $E$ has $F$-WGHP (signed $F$-WGHP) then $E_D$ has $F$-WGHP (signed $F$-WGHP) for any diagonal matrix $D$.*

*Proof.* The proof is similar to that of Theorem 3.2 in [1].

**Corollary 3.1** *Suppose that $E$ does not $F$-WGHP (signed $F$-WGHP) and WGHP (signed WGHP). Let $D$ be the diagonal matrix with $d_k \neq 0$ down the diagonal. Then $E_D$ does not have $F$-WGHP (signed $F$-WGHP) and WGHP (signed WGHP).*

*Proof.* If $D$ be the diagonal matrix with $d_k \neq 0$ down the diagonal, then $D^{-1}$ be the diagonal matrix with $1/d_k \neq 0$ down the diagonal. So if $E_D$ has, for example, $F$-WGHP, then $E = (E_D)_{D^{-1}}$ would have $F$-WGHP by Theorem 3.1, which contradicts the assumption of $E$. For the case of other properties, the proof is similar.

Since $bs$ does not have WGHP and signed $F$-WGHP, $cs$ does not have $F$-WGHP, and $c$ does not have signed WGHP, we have the following

**Corollary 3.2** *Let $D$ be the same as in Corollary* 3.1*. Then $bs_D$ does not have WGHP and signed $F$-WGHP, $cs_D$ does not have $F$-WGHP, and $c_D$ does not have signed WGHP.*

**Example 3.2** Note that $c_0$ is a monotone space, so it has WGHP, signed WGHP, signed $F$-WGHP and $F$-WGHP. But $(c_0)_{D_0}$ (in Example 2.5, when $n$=0, $D_n$ becomes $D_0$) has none of the four GHPs. It is clear that $F$-WGHP and signed $F$-WGHP imply WGHP and signed WGHP respectively. So if $(c_0)_{D_0}$ does not have WGHP (signed WGHP), then it does not have $F$-WGHP (signed $F$-WGHP). Thus it is sufficient to check that $(c_0)_{D_0}$ does not have $F$-WGHP and signed $F$-WGHP. It is obvious that

$$x = \{1,\ \frac{1}{2},\ \cdots,\ \frac{1}{k},\ \cdots\} \in (c_0)_{D_0},$$

since

$$D_0 x = (1,\ 0,\ 0,\ \cdots) \in c_0.$$

But if $(I_k)$ is an increasing sequence of intervals and $(n_k)$ is an arbitrary subsequence of $\{k\}$, then

$$\tilde{x} = \sum_{k=1}^{\infty} \chi_{I_{n_k}} x \notin (c_0)_{D_0},$$

since $D_0\tilde{x}$ has infinite many zeros and ones, i.e., $D_0\tilde{x} \notin c_0$. Thus it is to say that the conclusion is true for WGHP. In the case of signed WGHP, $x$, $(I_k)$ and $(n_k)$ are the same as described above. Let $(s_k)$ be a sequence of signs. Then

$$\tilde{y} = \sum_{k=1}^{\infty} s_{n_k} \chi_{I_{n_k}} x \notin (c_0)_{D_0}$$

since $D_0\tilde{y}$ has infinite many zeros, ones, and negative ones, i.e., $D_0\tilde{y} \notin c_0$.

**Example 3.3** Note that $l^\infty$ is a monotone space, so it has signed $F$-WGHP, but $l^\infty_B = bs$ does not have signed $F$-WGHP. So Theorem 3.1 does not hold for arbitrary triangular matrices. On the contrary, though $bs$ does not have signed $F$-WGHP, $bs_{B_1} = l^\infty$ has signed $F$-WGHP.

We next consider the case when the space $E$ has a locally convex Hausdorff topology under which $E$ is a $K$-space, i.e., under which the coordinate functionals

$$x = (x_k) \to x_k$$

are continuous. We say that $E$ has the property $AK$ if each $p_n$ is continuous and

$$p_n x \to x, \qquad \forall x \in E,$$

in which $p_n$ is the section map $E \to E$ which sends $x = (x_1,\ x_2,\ \cdots)$ to $(x_1,\ x_2,\ \cdots,\ x_n,\ 0,\ \cdots)$. Let $\mathcal{P}$ be a family of semi-norms which generates the topology of $E$. We give $E_A$ the locally convex topology generated by the semi-norms

$$p_A(x) = p(Ax), \qquad p \in \mathcal{P}$$

and

$$p_k(x) = |x_k|, \qquad k \in N.$$

Note that $E_A$ is a $K$-space and $A : E_A \to E$ is a continuous linear map.

In [1], it is shown that for any $p$ satisfying (2.1) the space $(c_0)_{R_p}$ has WGHP. Boos did this by showing that $(c_0)_{R_p}$ is an $FK$-$AK$-space (see its definition in [5], p. 357) and any such space has WGHP (see [7], Theorem 3.31d and also [8], Theorem 3.1 where it is shown that $FK$-$AK$-spaces enjoy even the so-called absolute strong P_GHP which is essentially stronger than WGHP).

**Remark 3.2**[6] In general, there is no comparison between WGHP and $AK$. For example, let $E = l_\infty$ with the sup-norm. Then $E$ has WGHP but not $AK$. While $(c,\ \omega(c,\ l^1))$ has $Ak$ but does not have WGHP, since scalar sequence space $\lambda$ is $AK$ for $\omega(\lambda,\ \lambda^\beta)$.

Let $\omega$ be the space of all scalar sequences

$$x = (x_k)$$

and

$$\mathcal{H} := \{h \in \omega | h_k = 0 \text{ or } h_k = 1 \text{ for all } k\}.$$

Let

$$\mathcal{H}_\phi = \mathcal{H} \bigcap c_{00}.$$

Then the set $\mathcal{H}_\phi$ is a directed set under the relation

$$h'' \succ h'$$

defined by

$$h''_k \geq h'_k, \qquad \forall k.$$

The space $E$ is said to have the property $UAK$ if for each

$$x = (x_k),$$

the net

$$h \cdot x = (h_k x_k),$$

where $h = (h_k)$ ranges over $\mathcal{H}_\phi$, converges to $x$ under the topology of $E$.

**Proposition 3.1** *$FK$-$UAK$ space $E$ has the $F$-WGHP.*

*Proof.* Suppose $\sigma_k$ be an increasing sequence of finite subset of $F$ and $x \in E$. Then there is a subsequence $n_k$ such that $\{\chi_{\cup\sigma_{n_k}} x : k \in N\}$ is a Cauchy sequence with the assumption that this space has the $UAK$-property. So when $k \to \infty$, there exists another element $\widetilde{x}$, such that

$$\chi_{\cup\sigma_{n_k}} x = \sum_{n=1}^{\infty} \chi_{\sigma_{n_k}} x = \widetilde{x} \in E$$

since the space $E$ is an $FK$ space, i.e., $E$ has $F$-WGHP.

**Example 3.4** For each nonnegative integer $n$, $(c_0)_{C_n}$ does not have $UAK$-property and $F$-WGHP.

It is obvious that

$$x = \{1,\ -1,\ \cdots,\ 1,\ -1,\ \cdots\} \in (c_0)_{C_n},$$

since

$$C_n x = \{(-1)^{n+1},\ 0,\ \frac{(-1)^{n+3}}{3},\ 0,\ \cdots,\ \frac{(-1)^{n+k}}{k},\ 0,\ \cdots\} \in c_0.$$

Set

$$\sigma_0 = \{1\}, \quad \sigma_1 = \{3,\ 5,\ 7\}, \quad \cdots$$

where $\sigma_k$ consists of $3^k$ $(k = 0, 1, 2, \cdots)$ consecutive odd integers. Let

$$\widetilde{x} = \sum_{k=0}^{\infty} \chi_{\sigma_k} x.$$

In the following we show that $\widetilde{x} \notin (c_0)_{C_n}$. In fact, we have

$$C_n \widetilde{x} = \Big\{\frac{1}{n+1},\ \frac{1}{n+2},\ \frac{2}{n+3},\ \frac{2}{n+4},\ \cdots,\ \frac{i}{n+2i-1},\ \frac{i}{n+2i},\ \cdots\Big\},$$

which implies that $C_n \widetilde{x} \notin c_0$ since

$$\frac{i}{(n+2i)} \to \frac{1}{2}, \qquad i \to \infty.$$

In the same time, for each $j \in N$, let

$$x^k = \chi_{\sigma_k} x, \qquad p = 3^k.$$

We have

$$C_n x^k = \Big\{0,\ \cdots,\ 0,\ \frac{1}{p+n+1},\ \frac{1}{p+n+2},\ \frac{2}{p+n+3},\ \frac{2}{p+n+4},\ \cdots,\ \frac{p}{3p+n},\ \frac{p}{p+n+1},\ 0,\ \cdots\Big\}.$$

This implies that

$$\|x^k\|_{C_n} = \|C_n x^k\| \nrightarrow 0$$

since

$$\frac{p}{p+n+1} = \frac{3^k}{3^{k+1}+n+1} \to \frac{1}{3}, \qquad k \to \infty.$$

So $(c_0)_{C_n}$ does not have $F$-WGHP and $UAK$-property.

The following two problems were presented in [1]: For what further classes of (triangular) matrices $A$, does $(c_0)_A$ have the $AK$-property (WGHP)? If $E$ has WGHP, does $E_{C_0}$ (or even $E_{R_p}$) have WGHP? The remainder of this section is devoted to solve these two problems.

**Theorem 3.2** *For each nonnegative integer $n$, $(c_0)_{C_n}$ has WGHP.*

*Proof.* $C_n$ is defined in Example 2.2 and $(c_0)_{C_n}$ is an $FK$-space under the semi-norm

$$x \to \|x\|_{C_n} = \|C_n x\|_\infty,$$

where $\|\cdot\|_\infty$ is the supremum norm, and the semi-norms $q_k$ defined by

$$q_k(x) := |x|.$$

So it suffices to show that $(c_0)_{C_n}$ has the $AK$-property which is equivalent to

$$x - \sum_{j=1}^{n-1} x_k e^k \to 0$$

with respect to $\|C_n x\|_\infty$, where $e^k$ is the canonical unit vector with a 1 in the $k$th coordinate and 0 in the other coordinates.

Let $x \in (c_0)_{C_n}$. It suffices to show that

$$t^n = x - \sum_{j=1}^{n-1} x_k e^k \to 0$$

with respect to $\|C_n x\|_\infty$, that is,

$$\sup_{r\geq n}\Big|\frac{1}{r}\sum_{k=n+p}^{r+p} x_k\Big| \to 0 \qquad (n\to\infty).$$

Let $\epsilon > 0$. Since $C_n x \in c_0$, there exists $N$ such that

$$\frac{1}{m}\Big|\sum_{k=p}^{m+p} x_k\Big| < \frac{\epsilon}{2}, \qquad m \geq N.$$

Suppose that $n > N$, $r \geq n$. Then

$$\begin{aligned}\frac{1}{r}\Big|\sum_{k=n+p}^{r+p} x_k\Big| &\leq \frac{1}{r}\Big|\sum_{k=p+1}^{r+p} x_k\Big| + \frac{1}{r}\Big|\sum_{k=p+1}^{n+p-1} x_k\Big| \\ &< \frac{\epsilon}{2} + \frac{1}{n-1}\Big|\sum_{k=p+1}^{n+p-1} x_k\Big| \\ &< \epsilon.\end{aligned}$$

Hence,

$$\|t^n\|_{C_n} \leq \epsilon, \qquad n > N.$$

Obviously, the following proposition is easy to prove.

**Proposition 3.2** *Let $\{A_n\}$ be a sequence of matrices. If for each $n$, $E$ and $E_{A_n}$ have WGHP, then $\bigcap E_{A_n}$ and $\bigcup E_{A_n}$ also have WGHP.*

The space $f$ was introduced by Lorentz[9]. We say that $(x_k) \in f$ if and only if there exists $l \in C$ such that

$$\frac{1}{r}\Big|\sum_{k=p+1}^{p+r} x_k\Big| \to l \qquad (r\to\infty,\ \text{uniformly in } p\geq 0).$$

If we take $l$ in the above equal to 0 only, then $f$ becomes the almost-null sequence space $f_0$. We have

$$c_0 \subset f_0 \subset f \subset l^\infty$$

with strict inclusions, and $c_0$, $f_0$, $f$ are closed subspaces of $l^\infty$, which is a Banach space with

$$\|x\| = \sup|x_k| \qquad \text{for each } x = (x_k) \in l^\infty.$$

But it is interest that $f_0$ can have the following representation

$$f_0 = \bigcap_{n=0}^{\infty} (c_0)_{C_n}.$$

Thus we can obtain the WGHP of $f_0$ by Theorem 3.2 and Proposition 3.2.

**Corollary 3.3** *The space $f_0$ has WGHP.*

Though the space $f_0$ has WGHP, we have the following result about $F$-WGHP and $UAK$-property of $f_0$ by the Example 3.4 and Proposition 3.2.

**Proposition 3.3** *The space $f_0$ does not have the $F$-WGHP and $UAK$-property.*

**Theorem 3.3** *If the local convex space $E$ is an $FK$-$AK$-space and $E_{C_n} \subseteq bs$, then for each nonnegative integer $n$, $E_{C_n}$ has the $FK$-$AK$-property.*

*Proof.* Because $E$ is a local convex and $FK$-space, $E$ is considered to be a separate complete paranormed space, and $\|\cdot\|$ is paranorm of $E$ and

$$\|x\|_{C_n} = \|C_n x\|$$

is the paranorm of $E_{C_n}$.

It suffices to show that

$$t^n = x - \sum_{j=1}^{n-1} x_k e^k \to 0$$

with respect to $\|\cdot\|_{C_n}$, that is,

$$\begin{aligned} &\|(0, \cdots, 0, x_n, x_{n+1}, \cdots)\|_{C_n} \\ = &\|(0, \cdots, 0, \frac{x_{n+p}}{n}, \frac{x_{n+p}+x_{n+p+1}}{n+1}, \cdots)\| \to 0 \qquad (n \to \infty). \end{aligned}$$

Since $x \in E_{C_n} \subseteq bs$ and $E$ has $AK$-property, so we have

$$\|(0, \cdots, 0, \frac{1}{n}\sum_{k=p+1}^{n+p} x_k, \frac{1}{n+1}\sum_{k=p+1}^{n+p+1} x_k, \cdots)\| \to 0 \qquad (n \to \infty).$$

In the same time,

$$\|(0, \cdots, 0, \frac{1}{n}\sum_{k=p+1}^{n+p-1} x_k, \frac{1}{n+1}\sum_{k=p+1}^{n+p-1} x_k, \cdots)\| \to 0 \qquad (n \to \infty).$$

Thus when $n \to \infty$

$$\begin{aligned} &\|(0, \cdots, 0, x_n, x_{n+1}, \cdots)\|_{C_n} \\ = \;&\|(0, \cdots, 0, \frac{1}{n}\sum_{k=p+1}^{n+p} x_k, \frac{1}{n+1}\sum_{k=p+1}^{n+p+1} x_k, \cdots) \\ &- (0, \cdots, 0, \frac{1}{n}\sum_{k=p+1}^{n+p-1} x_k, \frac{1}{n+1}\sum_{k=p+1}^{n+p-1} x_k, \cdots)\| \\ \le \;&\|(0, \cdots, 0, \frac{1}{n}\sum_{k=p+1}^{n+p} x_k, \frac{1}{n+1}\sum_{k=p+1}^{n+p+1} x_k, \cdots)\| \\ &+ \|(0, \cdots, 0, \frac{1}{n}\sum_{k=p+1}^{n+p-1} x_k, \frac{1}{n+1}\sum_{k=p+1}^{n+p-1} x_k, \cdots)\| \to 0. \end{aligned}$$

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