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Existence of Solutions to Generalized Vector Quasi-variational-like Inequalities with Set-valued Mappings

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Abstract: In this paper, we introduce and study a class of generalized vector quasi-variational-like inequality problems, which includes generalized nonlinear vector variational inequality problems, generalized vector variational inequality problems and generalized vector variational-like inequality problems as special cases. We use the maximal element theorem with an escaping sequence to prove the existence results of a solution for generalized vector quasi-variational-like inequalities without any monotonicity conditions in the setting of locally convex topological vector space.

Key words: generalized vector quasi-variational-like inequality, maximal element theorem, upper semicontinuous diagonal convexity, locally convex topological vector space

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1 Introduction

Vector variational inequality was first introduced and studied by Giannessi[1] in the setting of finite-dimensional Euclidean spaces. This is a generalization of a scalar variational inequality to the vector case by virtue of multi-criteria consideration. Throughout the development over the last twenty years, existence theorems of solutions of vector variational inequalities in various situations have been studied by many authors (see, for example, [2–5] and the references therein). Recently, Peng and Rong[6], Ahmad and Irfan[7] and Xiao *et al.*[8]

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proved some existence theorems of solutions to a class of generalized nonlinear variational inequalities.

In this paper, we introduce a new class of generalized vector quasi-variational-like inequality problems and utilize the maximal element theorem with an escaping sequence to prove the existence of its solutions in the setting of locally convex topological vector spaces (locally convex spaces, in short). Some results of [6–8] are improved and extended.

2 Preliminaries

Let Z be a locally convex space and X be a nonempty convex subset of a Hausdorff topological vector space E (t.v.s., in short). We denote by $L(E,Z)$ the space of all continuous linear operators from E into Z and by $\langle l, x\rangle$ the evaluation of $l \in L(E,Z)$ at $x \in E$. Let $L(E,Z)$ be a space equipped with σ-topology. By the corollary of Schaefer (see page 80 in [9]), $L(E,Z)$ becomes a locally convex space. By Ding and Tarafdar[10], the bilinear mapping $\langle\, \cdot\, ,\, \cdot\, \rangle : L(E,Z)\times E \to Z$ is continuous.

Let int$S$ and co$S$ denote the interior and convex hull of a set S, respectively, $C: X \to 2^Z$ be a set-valued mapping such that $\mathrm{int}C(x) \neq \emptyset$ for each $x \in X$, and $\eta : X \times X \to E$ be a vector-valued mapping. Let $T : X \to 2^{L(E,Z)}$, $D : X \to 2^X$, $A : L(E,Z) \to 2^{L(E,Z)}$ and $H : X \times X \to 2^Z$ be four set-valued mappings. We consider the following generalized vector quasi-variational-like inequality problem (GVQVLIP, in short):

Find $\bar{x} \in X$ such that $\bar{x} \in D(\bar{x})$ and for all $y \in D(\bar{x})$, there exists $\bar{s} \in T(\bar{x})$ satisfying

$$\langle A\bar{s},\, \eta(y,\bar{x})\rangle + H(\bar{x},y) \nsubseteq -\mathrm{int}C(\bar{x}). \tag{2.1}$$

The following problems are special cases of GVQVLIP.

(i) For all $x \in X$, if $D(x) = X$, then (2.1) reduces to

$$\langle A\bar{s},\ \eta(y,\bar{x})\rangle + H(\bar{x},y) \nsubseteq -\mathrm{int}C(\bar{x}), \qquad y \in X, \tag{2.2}$$

which has been studied by Xiao *et al.*[8]

Find $\bar{x} \in X$, such that there exists $\bar{s} \in T(\bar{x})$ satisfying (2.2).

(ii) Let $A = I$ be a single-valued mapping and $H \equiv 0$. Then (2.1) reduces to

$$\langle \bar{v},\ \eta(y,\bar{x})\rangle \notin -\mathrm{int}C(\bar{x}), \tag{2.3}$$

which has been studied by Peng and Rong[6].

Find $\bar{x} \in X$, such that $\bar{x} \in D(\bar{x})$ and for all $y \in D(\bar{x})$, there exists $\bar{v} \in T(\bar{x})$ satisfying (2.3).

For suitable and appropriate choice of the mappings D, T, A, H, η, one can obtain various new and previously known variational inequality problems as special cases (see [6], [8] and the references therein).

In order to prove the main results, we need the following definitions and lemmas.

Let X be a topological space. A subset S of X is said to be compactly open (respectively, compactly closed) in X if for any nonempty compact subset K of X, $S \bigcap K$ is open (respectively, closed) in S. Let Y be a topological space and $T : X \to 2^Y$ be a set-valued mapping. Then, T is said to be open valued if the set $T(x)$ is open in X for each $x \in X$. T is said to have open lower sections if T^{-1} is open valued, i.e., the set $T^{-1}(y) = \{x \in X : y \in T(x)\}$ is open in X for each $y \in Y$. T is said to be compactly open valued if the set $T(x)$ is compactly open in X for each $x \in X$, and T is said to have compactly open lower sections if T^{-1} is compactly open valued. Clearly, each open-valued (respectively, closed-valued) mapping $T : X \to 2^Y$ is compactly open-valued (respectively, compactly closed-valued). T is said to be upper semicontinuous, if for any $x_0 \in X$ and for each open set U in Y containing $T(x_0)$, there is a neighborhood V of x_0 in X such that $T(x) \subseteq U$, for all $x \in V$; T is said to be closed if the set $\{(x, y) \in X \times Y : y \in T(x)\}$ is closed in $X \times Y$.

Definition 2.1[8,11–12] *Let K be a convex subset of a t.v.s. E and Z be t.v.s. Let $C : K \to 2^Z$ be a set-valued mapping. Assume given any finite subset $\Lambda = \{x_1, x_2, \cdots, x_n\}$ of X, any $x = \sum\limits_{i=1}^{n} \alpha_i x_i$ with $\alpha_i \geq 0$ for $i = 1, 2, \cdots, n$, and $\sum\limits_{i=1}^{n} \alpha_i = 1$. Then,*

(i) *a single-valued mapping $f : K \times K \to Z$ is said to be vector 0-diagonally convex in the second argument if*

$$\sum_{i=1}^{n} \alpha_i f(x, x_i) \notin -\mathrm{int}C(x);$$

(ii) *a set-valued mapping $f : K \times K \to 2^Z$ is said to be generalized vector 0-diagonally convex in the second argument if*

$$\sum_{i=1}^{n} \alpha_i f(x, x_i) \not\subseteq -\mathrm{int}C(x).$$

Lemma 2.1[13] *Let X and Y be two topological spaces. If $T : X \to 2^Y$ is an upper semicontinuous set-valued mapping with closed values, then T is closed.*

Lemma 2.2[14] *Let X and Y be two topological spaces, and $T : X \to 2^Y$ be an upper semicontinuous set-valued mapping with compact values. Suppose that $\{x_\alpha\}$ is a net in X such that $x_\alpha \to x_0$. If $y_\alpha \in T(x_\alpha)$ for each α, then there is a $y_0 \in T(x_0)$ and a subset y_β of y_α such that $y_\beta \to y_0$.*

Lemma 2.3[15] *Let X and Y be two topological spaces. Suppose that $T : X \to 2^Y$ is a set-valued mapping having open lower sections. Then the set-valued mapping $F : X \to 2^Y$ defined by that for each $x \in X$, $F(x) = \mathrm{co}T(x)$ has open lower sections.*

Definition 2.2[16] *Let E be a topological space and X be a subset of E such that $X = \bigcup\limits_{n=1}^{\infty} X_n$, where $\{X_n\}_{n=1}^{\infty}$ is an increasing (in the sense that $X_n \subseteq X_{n+1}$) sequence of nonempty compact sets. A sequence $\{x_n\}_{n=1}^{\infty}$ in X is said to be an escaping sequence from X (relative to $\{X_n\}_{n=1}^{\infty}$) if for each $n = 1, 2, \cdots$, there exists $m > 0$ such that $x_k \notin X_k$, for all $k \geq m$.*

Lemma 2.4[6,16] *Let E be a topological vector space and X be a subset of E such that $X = \bigcup\limits_{n=1}^{\infty} X_n$, where $\{X_n\}_{n=1}^{\infty}$ is an increasing sequence of nonempty compact sets of X. Assume that the set-valued mapping $S : X \to 2^X$ satisfies the following conditions:*

(i) *For each $x \in X$, $S^{-1}(x) \bigcap X_n$ is open in X_n for all $n = 1, 2, \cdots$;*

(ii) *For each* $x \in X$, $x \notin \mathrm{co}S(x)$;

(iii) *For each sequence* $\{x_n\}_{n=1}^{\infty}$ *in* X *with* $x_n \in X_n$ *for all* $n = 1, 2, \cdots$, *which is escaping from* X *relative to* $\{X_n\}_{n=1}^{\infty}$, *there exist* $n \in N$ *and* $y_n \in X_n$ *such that* $y_n \in S(x_n) \bigcap X_n$. *Then there exists an* $\bar{x} \in X$ *such that* $S(\bar{x}) = \emptyset$.

3 Existence Results

In this section, we prove some existence results of solutions for generalized vector quasi-variational-like inequalities without any monotonicity conditions in the setting of locally convex topological vector space.

Theorem 3.1 *Let* E *be a Hausdorff topological vector space,* X *be a subset of* E *such that* $X = \bigcup\limits_{n=1}^{\infty} X_n$, *where* $\{X_n\}_{n=1}^{\infty}$ *is an increasing sequence of nonempty, compact and convex subset of* X, *and* Z *be a locally convex space. Let* $L(E, Z)$ *be equipped with* σ*-topology. Let* $D : X \to 2^X$ *be a set-valued mapping with nonempty convex values and compactly open lower sections, the set* $W = \{x \in X : x \in D(x)\}$ *be closed,* $C : X \to 2^Z$ *be a set-valued mapping such that* $C(x)$ *is a closed pointed and convex cone with* $\mathrm{int}C(x) \neq \emptyset$ *for each* $x \in X$, *and the set-valued mapping* $M = Z \setminus \{-\mathrm{int}C(x)\}$ *be upper semicontinuous on* X. *Let* $T : X \to 2^{L(E,Z)}$ *be upper semicontinuous on* X *with compact values and* $H : X \times X \to 2^Z$ *be generalized vector* 0*-diagonally convex in the second argument. Let* $\eta : X \times X \to E$ *be affine in the first argument with* $\eta(x, x) = 0$ *for all* $x \in X$. *For each* $y \in X$, *assume that* $\langle A(\,\cdot\,),\ \eta(y,\,\cdot\,)\rangle + H(\,\cdot\,, y) : L(E, Z) \times X \times X \to 2^Z$ *is an upper semicontinuous set-valued mapping with compact values. Suppose that*

(A_1) *for each sequence* $\{x_n\}_{n=1}^{\infty}$ *in* X *with* $x_n \in X_n$ *for all* $n = 1, 2, \cdots$, *which is escaping from* X *relative to* $\{X_n\}_{n=1}^{\infty}$, *there exist* $m \in N$ *and* $z_m \in D(x_m) \bigcap X_m$ *such that for all* $s_m \in T(x_m)$,

$$\langle As_m,\ \eta(z_m, x_m)\rangle + H(x_m, z_m) \subseteq -\mathrm{int}C(x_m).$$

Then GVQVLIP has a solution.

Proof. Define a set-valued mapping $P : X \to 2^X$ by setting

$$P(x) = \{y \in X : \langle As,\ \eta(y, x)\rangle + H(x, y) \subseteq -\mathrm{int}C(x),\ s \in T(x)\}, \qquad x \in X.$$

We first prove that $x \notin \mathrm{co}P(x)$ for all $x \in X$. To see this, by way of contradiction, assume that there existed some point $\bar{x} \in X$ such that $\bar{x} \in \mathrm{co}P(\bar{x})$. Then there would exist a finite subset $\{y_1, y_2, \cdots, y_n\}$ of X such that for $\bar{x} \subseteq \mathrm{co}\{y_1, y_2, \cdots, y_n\}$ we have

$$\langle As,\ \eta(y_i, \bar{x})\rangle + H(\bar{x}, y_i) \subseteq -\mathrm{int}C(\bar{x}), \qquad i = 1, 2, \cdots, n.$$

Since $\mathrm{int}C(\bar{x})$ is a convex set and η is affine in the first argument, for $i = 1, 2, \cdots, n, \alpha_i \geq 0$ with $\sum\limits_{i=1}^{n} \alpha_i = 1,\ \bar{x} = \sum\limits_{i=1}^{n} \alpha_i y_i$, we have

$$\Big\langle As,\ \eta\Big(\sum_{i=1}^{n} \alpha_i y_i,\ \bar{x}\Big)\Big\rangle + \sum_{i=1}^{n} \alpha_i H(\bar{x}, y_i) \subseteq -\mathrm{int}C(\bar{x}).$$

Since $\eta(\bar{x},\bar{x}) = 0$, we have $\sum\limits_{i=1}^{n} \alpha_i H(\bar{x}, y_i) \subseteq -\mathrm{int}C(\bar{x})$, which contradicts the fact that H satisfies the generalized vector 0-diagonal convexity in the second argument. Therefore, $x \notin \mathrm{co}P(x)$ for all $x \in X$.

We also define a set-valued mapping $G : X \to 2^X$ by

$$G(x) = \begin{cases} D(x) \bigcap \mathrm{co}P(x), & x \in W; \\ D(x), & x \in X \setminus W. \end{cases}$$

Then, for each $x \in X,\ G(x)$ is convex. Suppose that there exists $\bar{x} \in X$ such that $\bar{x} \in G(\bar{x})$. If $\bar{x} \in W$, then $\bar{x} \in D(x) \cap \mathrm{co}P(x)$, which contradicts $x \notin \mathrm{co}P(x)$ for all $x \in X$. If $\bar{x} \notin W$, then $G(\bar{x}) = D(\bar{x})$ which implies $\bar{x} \in D(\bar{x})$, a contradiction. Hence,

$$x \notin G(x) = \mathrm{co}G(x), \qquad x \in X,$$

and the condition (ii) of Lemma 2.4 is satisfied.

Next, we prove that the set

$$P^{-1}(y) = \{x \in X : \langle As,\ \eta(y,x)\rangle + H(x,y) \subseteq -\mathrm{int}C(x),\ s \in T(x)\}$$

is open for each $y \in X$. That is, P has open lower sections in X. Consider the set

$$(P^{-1}(y))^C = \{x \in X : \{\langle As,\ \eta(y,x)\rangle + H(x,y)\} \bigcap Z \backslash \{-\mathrm{int}C(x)\} \neq \emptyset,\ \exists s \in T(x)\},$$

which is the complement of $P^{-1}(y)$. We only need to prove that $(P^{-1}(y))^C$ is closed for all $y \in X$. Let $\{x_\alpha\}$ be a net in $(P^{-1}(y))^C$ such that $x_\alpha \to x^*$. Then there exists an $s_\alpha \in T(x_\alpha)$ such that

$$\{\langle As_\alpha,\ \eta(y,x_\alpha)\rangle + H(x_\alpha,y)\} \bigcap Z \backslash \{-\mathrm{int}C(x_\alpha)\} \neq \emptyset.$$

Since $T : X \to 2^{L(E,Z)}$ is an upper semicontinuous set-valued mapping with compact values, by Lemma 2.2, $\{s_\alpha\}$ has a convergent subset with limit, say s^*, and $s^* \in T(x^*)$. Without loss of generality, we may assume that $s_\alpha \to s^*$. Suppose that

$$z_\alpha \in \{\langle As_\alpha,\ \eta(y,x_\alpha)\rangle + H(x_\alpha,y)\} \bigcap Z \backslash \{-\mathrm{int}C(x_\alpha)\}.$$

Since $\langle A(\ \cdot\ ),\ \eta(y,\ \cdot\ )\rangle + H(\ \cdot\ ,y)$ is upper semicontinuous with compact values, by Lemma 2.2, there exist a $z^* \in \langle As^*,\ \eta(y,x^*)\rangle + H(x^*,y)$ and a subset $\{z_\beta\}$ of $\{z_\alpha\}$ such that $z_\beta \to z^*$.

On the other hand, since $Z \setminus \{-\mathrm{int}C(x)\}$ is upper semicontinuous with closed values, by Lemma 2.1, $z^* \in Z \setminus \{-\mathrm{int}C(x^*)\}$. Hence,

$$\{\langle As^*,\ \eta(y,x^*)\rangle + H(x^*,y)\} \bigcap Z \backslash \{-\mathrm{int}C(x^*)\} \neq \emptyset.$$

Thus, $(P^{-1}(y))^C$ is closed in X. Therefore, P has open lower sections in X. By Lemma 2.3, $\mathrm{co}P^{-1}(y)$ is also open for each $y \in X$. Since $D^{-1}(y)$ is compactly open for each $y \in X$,

$$\begin{aligned} &G^{-1}(y) \\ &= \{x \in X : y \in G(x)\} \\ &= \{x \in W : y \in [D(x) \textstyle\bigcap \mathrm{co}P(x)]\} \textstyle\bigcap \{x \in X \backslash W : y \in D(x)\} \\ &= (W \textstyle\bigcap D^{-1}(y) \textstyle\bigcap \mathrm{co}P^{-1}(y)) \textstyle\bigcup [(X \backslash W) \textstyle\bigcap D^{-1}(y)] \\ &= [(W \textstyle\bigcap D^{-1}(y) \textstyle\bigcap \mathrm{co}P^{-1}(y)) \textstyle\bigcup (X \backslash W)] \textstyle\bigcap [(W \textstyle\bigcap D^{-1}(y) \textstyle\bigcap \mathrm{co}P^{-1}(y)) \textstyle\bigcup D^{-1}(y)] \\ &= \{X \textstyle\bigcap [(D^{-1}(y) \textstyle\bigcap \mathrm{co}P^{-1}(y)) \textstyle\bigcup (X \backslash W)]\} \textstyle\bigcap [(W \textstyle\bigcup D^{-1}(y)) \textstyle\bigcap (D^{-1}(y))] \end{aligned}$$

$= [(D^{-1}(y)\bigcap \mathrm{co}P^{-1}(y))\bigcup(X\backslash W)]\bigcap D^{-1}(y)$

$= (D^{-1}(y)\bigcap(\mathrm{co}P^{-1}(y)))\bigcup((X\backslash W)\bigcap(D^{-1}(y))).$

Therefore, $G^{-1}(y)$ also has compactly open values in X for all $y \in X$, the condition (i) of Lemma 2.4 is satisfied. The condition (A_1), and implies the condition (iii) of Lemma 2.4. Therefore, by Lemma 2.4, there exists an $\bar{x} \in X$ such that $G(\bar{x}) \neq \emptyset$.

Since for each $x \in X$, $D(x)$ is nonempty, we have $\bar{x} \in D(\bar{x})$ such that $D(\bar{x})\bigcap \mathrm{co}P(\bar{x}) = \emptyset$, which implies that $\bar{x} \in D(\bar{x})$ such that $D(\bar{x})\bigcap P(\bar{x}) = \emptyset$, that is, $\bar{x} \in D(\bar{x})$, and for all $y \in D(\bar{x})$, there exists an $\bar{s} \in T(\bar{x})$ satisfying $\langle A\bar{s},\ \eta(y,\bar{x})\rangle + H(\bar{x},y) \subseteq -\mathrm{int}C(\bar{x})$.

Remark 3.1 If $D(x) = X$ for all $x \in X$, then by Theorem 3.1, we recover Theorem 3.1 in [8].

Theorem 3.2 *Let E be a Hausdorff topological vector space, X be a subset of E such that $X = \bigcup\limits_{n=1}^{\infty} X_n$, where $\{X_n\}_{n=1}^{\infty}$ is an increasing sequence of nonempty, compact and convex subset of X, Z be a locally convex space, and $L(E,Z)$ be equipped with σ-topology, $D : X \to 2^X$ be a set-valued mapping with nonempty convex values and compactly open lower sections, the set $W = \{x \in X : x \in D(x)\}$ be closed, $C : X \to 2^Z$ be a set-valued mapping such that $C(x)$ is a closed pointed and convex cone with $\mathrm{int}C(x) \neq \emptyset$ for each $x \in X$, the set-valued mapping $M = Z \setminus \{-\mathrm{int}C(x)\}$ be upper semicontinuous on X, $T : X \to 2^{L(E,Z)}$ be upper semicontinuous on X with compact values, and $\eta : X \times X \to E$ be affine in the first argument with $\eta(x,x) = 0$ for all $x \in X$. For each $y \in X$, assume that*

$$\langle A(\ \cdot\ ),\ \eta(y,\ \cdot\ )\rangle + H(\ \cdot\ ,y) : L(E,Z) \times X \times X \to 2^Z$$

is an upper semicontinuous set-valued mapping with compact values. Suppose that there exists a mapping $R : X \times X \to 2^Z$ such that

(i) *for all $x, y \in X$, there exists an $s \in T(x)$ such that*

$$R(x,y) - [\langle As,\ \eta(y,x)\rangle + H(x,y)] \subseteq -\mathrm{int}C(x);$$

(ii) *For any finite set $\{y_1, y_2, \cdots, y_n\} \subseteq X$ and $\bar{x} = \sum\limits_{j=1}^{n} \alpha_j y_j$ with $\alpha_j \geq 0$ and $\sum\limits_{j=1}^{n} \alpha_j = 1$, there is a $j \in \{1, 2, \cdots, n\}$ such that $R(\bar{x}, y_j) \nsubseteq -\mathrm{int}C(\bar{x})$;*

(iii) *For each sequence $\{x_n\}_{n=1}^{\infty}$ in X with $x_n \in X_n$ for all $n = 1, 2, \cdots$, which is escaping from X relative to $\{X_n\}_{n=1}^{\infty}$, there exist an $m \in N$ and a $z_m \in D(x_m) \cap X_m$ such that*

$$\langle As_m,\ \eta(z_m, x_m)\rangle + H(x_m,\ z_m) \subseteq -\mathrm{int}C(x_m), \qquad s_m \in T(x_m).$$

Then GVQVLIP has a solution.

Proof. Define two set-valued mappings $P : X \to 2^X, P_1 : X \to 2^X$ by

$$P(x) = \{y \in X : \langle As,\ \eta(y,x)\rangle + H(x,y) \subseteq -\mathrm{int}C(x),\ \forall s \in T(x)\}, \qquad x \in X,$$

$$P_1(x) = \{y \in X : R(x,y) \subseteq -\mathrm{int}C(x)\}, \qquad x \in X.$$

We first prove that $x \notin \mathrm{co}(P_1(x))$ for all $x \in X$. To see this, by way of contradiction, assume that there existed some point $\bar{x} \in X$ such that $\bar{x} \in \mathrm{co}(P_1(\bar{x}))$. Then there exist finite points $y_1, y_2, \cdots, y_n$ in X and $\alpha_j \geq 0$ with $\sum\limits_{j=1}^{n} \alpha_j = 1$ such that $\bar{x} = \sum\limits_{j=1}^{n} \alpha_j y_j$ and $y_j \in P_1(\bar{x})$ for all $j = 1, 2, \cdots, n$. That is, $R(\bar{x}, y_j) \subseteq -\mathrm{int}C(\bar{x})$, $j = 1, 2, \cdots, n$. This contradicts the condition (ii). Therefore, $x \notin \mathrm{co}(P_1(x))$ for all $x \in X$.

The condition (i) implies that $P_1(x) \supseteq P(x)$ for all $x \in X$. Hence, $x \notin \mathrm{co}(P(x))$ for all $x \in X$.

The remainder of the proof is the same as of Theorem 3.1.

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