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Coefficient Estimates for a Class of m-fold Symmetric Bi-univalent Function Defined by Subordination

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Abstract: In this paper, we investigate the coefficient estimates of a class of m-fold bi-univalent function defined by subordination. The results presented in this paper improve or generalize the recent works of other authors.

Key words: analytic function, univalent function, coefficient estimate, m-fold symmetric bi-univalent function, subordination

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1 Introduction

Let $\mathcal{A}$ denote the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk $U = \{z\colon |z| < 1\}$. We denote by $\mathcal{S}$ the class of all functions $f(z) \in \mathcal{A}$ which are univalent in U.

It is well known that every function $f \in \mathcal{S}$ has an inverse f^{-1}, defined by

$$f^{-1}(f(z)) = z \quad (z \in U)$$

and

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$$f(f^{-1}(\omega)) = \omega \qquad \left(|\omega| < r_0(f),\ r_0(f) \geq \frac{1}{4}\right).$$

The inverse functions $g = f^{-1}$ is given by

$$f^{-1}(\omega) = \omega - a_2\omega^2 + (2a_2^2 - a_3)\omega^3 - (5a_2^3 - 5a_2a_3 + a_4)\omega^4 + \cdots. \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in U if both $f(z)$ and $f^{-1}(z)$ are univalent in U. Let $\varSigma$ denote the class of all bi-univalent functions in unit disk U.

For each functions $f \in \mathcal{S}$, the function

$$h(z) = \sqrt{m} f(z^m) \qquad (z \in U,\ m \in \mathbf{N}^+)$$

is univalent and maps the unit disk U into a region with m-fold symmetry. A function is said to be m-fold symmetric (see [1] and [2]) if it has the following normalized form:

$$f(z) = z + \sum_{k=1}^{\infty} a_{mk+1} z^{mk+1} \qquad (z \in U,\ m \in \mathbf{N}^+). \tag{1.3}$$

Analogous to the concept of m-fold symmetric univalent functions, here we introduced the concept of m-fold symmetric bi-univalent functions. For the normalized form of f given by (1.3), Srivastava *et al.*[3] obtained the series expansion for f^{-1} as follows:

$$\begin{aligned} g(\omega) &= f^{-1}(\omega) \\ &= \omega - a_{m+1}\omega^{m+1} + [(m+1)a_{m+1}^2 - a_{2m+1}]\omega^{2m+1} \\ &\quad - \left[\frac{1}{2}(m+1)(3m+2)a_{m+1}^3 - (3m+2)a_{m+1}a_{2m+1} + a_{3m+1}\right]\omega^{3m+1} + \cdots. \end{aligned} \tag{1.4}$$

We denote by $\varSigma_m$ the class of m-fold symmetric bi-univalent function in U. For $m = 1$, the formula (1.4) coincides with the formula (1.2) of the class $\varSigma$. Some m-fold symmetric bi-univalent functions are given as follows:

$$\left(\frac{z^m}{1 - z^m}\right)^{\frac{1}{m}}, \qquad [-\log(1 - z^m)]^{\frac{1}{m}}, \qquad \left[\frac{1}{2}\log\left(\frac{1 + z^m}{1 - z^m}\right)\right]^{\frac{1}{m}}.$$

The class of bi-univalent functions was first introduced and studied by Lewin[4] and was showed that $|a_2| < 1.51$. Brannan and Clunie[5] improved Lewin's results to $|a_2| \leq \sqrt{2}$ and later Netanyahu[6] proved that $\max\{|a_2|\} = \frac{4}{3}$ if $f(z) \in \varSigma$. Recently, many authors investigated the estimates of the coefficients $|a_2|$ and $|a_3|$ for various subclasses of bi-univalent functions (see [7]–[9]). Not much is known about the bounds on general coefficient $|a_n|$ for $n \geq 4$. In the literature, only few works determine general coefficient bounds $|a_n|$ for the analytic bi-univalent functions (see [10]–[14]).

In this paper, let $\mathcal{P}$ denote the class of analytic functions of the form

$$p(z) = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \cdots,$$

and then

$$\mathrm{Re}\{p(z)\} > 0 \qquad (z \in U).$$

By [2], the m-fold symmetric function p in the class $\mathcal{P}$ is given of the form:

$$p(z) = 1 + p_m z + p_{2m} z^{2m} + p_{3m} z^{3m} + \cdots.$$

Throughout this paper, it is assumed that φ is an analytic function with positive real part in the unit disk U such that $\varphi(0) = 1$, $\varphi'(0) > 0$, and $\varphi(U)$ is symmetric with respect to the real axis. The function φ has a series expansion of the form:

$$\varphi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \cdots \qquad (B_1 > 0). \tag{1.5}$$

Let $u(z)$ and $v(z)$ be two analytic functions in the unit disk U with

$$u(0) = v(0) \qquad \text{and} \qquad \max\{|u(z)|, |v(z)|\} < 1.$$

We observe that

$$u(z) = b_m z^m + b_{2m} z^{2m} + b_{3m} z^{3m} + \cdots$$

and

$$v(z) = c_m z^m + c_{2m} z^{2m} + c_{3m} z^{3m} + \cdots .$$

We also observe that

$$|b_m| \le 1, \quad |b_{2m}| \le 1 - |b_m|^2, \quad |c_m| \le 1, \quad |c_{2m}| \le 1 - |c_m|^2. \tag{1.6}$$

Making some simple computations, we have

$$\varphi(u(z)) = 1 + B_1 b_m z^m + (B_1 b_{2m} + B_2 b_m^2) z^{2m} + \cdots \qquad (|z| < 1) \tag{1.7}$$

and

$$\varphi(v(w)) = 1 + B_1 c_m w^m + (B_1 c_{2m} + B_2 c_m^2) w^{2m} + \cdots \qquad (|w| < 1). \tag{1.8}$$

Recently, many researchers (e.g., [15]–[19]) have introduced and investigated a lot of interesting subclass of m-fold symmetric bi-univalent functions. Motivated by them, we investigate the estimates $|a_{m+1}|$ and $|a_{2m+1}|$ for function belonging to the new general subclass $\mathcal{H}_{\Sigma,m}(\varphi)$ of Σ_m. A new subclass $\mathcal{H}_{\Sigma,m}(\varphi)$ of Σ_m is defined as follows:

Definition 1.1[15] *A function $f \in \Sigma_m$ given by* (1.3) *is said to be in the class $\mathcal{H}_{\Sigma,m}(\varphi)$ if it satisfies*

$$f'(z) \prec \varphi(z) \qquad (z \in U),$$
$$g'(\omega) \prec \varphi(\omega) \qquad (\omega \in U),$$

where the function g is given by (1.4).

For various special choices of the function $\varphi(z)$ and for the case of $m = 1$, our function class $\mathcal{H}_{\Sigma,m}(\varphi)$ reduces the following known function classes.

(1) In the case of $m = 1$ in Definition 1.1, one has

$$\mathcal{H}_{\Sigma,m}(\varphi) = \mathcal{H}_{\Sigma,1}(\varphi) = \mathcal{H}_{\Sigma}(\varphi)$$

studied by Ali *et al.*[13].

(2) In the case of $m = 1$ and $\varphi(z) = \left(\frac{1+z}{1-z}\right)^{\gamma}$ $(0 < \gamma \le 1)$ in Definition 1.1, one has

$$\mathcal{H}_{\Sigma,m}(\varphi) = \mathcal{H}_{\Sigma,1}\left(\left(\frac{1+z}{1-z}\right)^{\gamma}\right)$$

studied by Srivastava *et al.*[14].

(3) In the case of $m = 1$ and $\varphi(z) = \frac{1+(1-2\gamma)z}{1-z}$ $(0 \le \gamma < 1)$ in Definition 1.1, one has

$$\mathcal{H}_{\Sigma,m}(\varphi)=\mathcal{H}_{\Sigma,1}\left(\frac{1+(1-2\gamma)z}{1-z}\right)$$

studied by Srivastava *et al.*[14].

(4) In the case of $\varphi(z)=\left(\frac{1+z}{1-z}\right)^{\alpha}$ $(0<\alpha\le 1)$ in Definition 1.1, one has

$$\mathcal{H}_{\Sigma,m}(\varphi)=\mathcal{H}_{\Sigma,m}\left(\left(\frac{1+z}{1-z}\right)^{\alpha}\right)=\mathcal{H}^{\alpha}_{\Sigma_m}$$

investigated by Srivastava *et al.*[19].

(5) In the case of $\varphi(z)=\frac{1+(1-2\beta)z}{1-z}$ $(0\le\beta<1)$ in Definition 1.1, one has

$$\mathcal{H}_{\Sigma,m}(\varphi)=\mathcal{H}_{\Sigma,m}\left(\frac{1+(1-2\beta)z}{1-z}\right)=\mathcal{H}^{\beta}_{\Sigma_m}$$

investigated by Srivastava *et al.*[19].

2 Coefficient Estimates

Theorem 2.1 *Let $f(z)$ given by* (1.3) *be in the class $\mathcal{H}_{\Sigma,m}(\varphi)$. Then*

$$|a_{m+1}|\le\min\left\{\frac{B_1}{m+1},\ \sqrt{\frac{2B_1+2|B_2|}{(2m+1)(m+1)}},\ \Omega_1\right\},\tag{2.1}$$

$$|a_{2m+1}|\le\begin{cases}\dfrac{B_1}{2m+1}, & B_1<\dfrac{2(m+1)}{2m+1};\\[2ex] \min\left\{\dfrac{B_1^2}{2(m+1)},\ \left(1-\dfrac{2(m+1)}{(2m+1)B_1}\right)\dfrac{B_1+|B_2|}{2m+1}+\dfrac{B_1}{2m+1},\ \Omega_2\right\}, & B_1\ge\dfrac{2(m+1)}{2m+1},\end{cases}\tag{2.2}$$

where

$$\Omega_1=\frac{B_1\sqrt{2B_1}}{\sqrt{(m+1)[2(m+1)B_1+|(2m+1)B_1^2-2(m+1)B_2|]}},$$

$$\Omega_2=\left(1-\frac{2(m+1)}{(2m+1)B_1}\right)\frac{B_1^3}{2(m+1)B_1+|(2m+1)B_1^2-2(m+1)B_2|}+\frac{B_1}{2m+1}.$$

Proof. Let $f\in\mathcal{H}_{\Sigma,m}(\varphi)$ and $g=f^{-1}$. Then there are analytic functions $u\colon U\to U$, and $v\colon U\to U$ with $u(0)=v(0)=0$ satisfying the following conditions:

$$f'(z)=\varphi(u(z)),\qquad g'(\omega)=\varphi(v(\omega)).\tag{2.3}$$

Since

$$f'(z)=1+(m+1)a_{m+1}z^m+(2m+1)a_{2m+1}z^{2m}+\cdots$$

and

$$g'(\omega)=1-(m+1)a_{m+1}\omega^m+(2m+1)[(m+1)a_{m+1}^2-a_{2m+1}]\omega^{2m}+\cdots,$$

it follows from (1.7), (1.8) and (2.3) that

$$(m+1)a_{m+1}=B_1b_m,\tag{2.4}$$

$$(2m+1)a_{2m+1} = B_1 b_{2m} + B_2 b_m^2, \tag{2.5}$$

$$-(m+1)a_{m+1} = B_1 c_m, \tag{2.6}$$

$$(2m+1)(m+1)a_{m+1}^2 - (2m+1)a_{2m+1} = B_1 c_{2m} + B_2 c_m^2. \tag{2.7}$$

From (2.4) and (2.6), we find that

$$b_m = -c_m, \tag{2.8}$$

$$a_{m+1}^2 = \frac{B_1^2(b_m^2 + c_m^2)}{2(m+1)^2}. \tag{2.9}$$

By using the inequalities given by (1.6) in (2.9) for the coefficients b_m and c_m, we obtain

$$|a_{m+1}| \leq \frac{B_1}{m+1}. \tag{2.10}$$

Adding (2.5) to (2.7), we have

$$(2m+1)(m+1)a_{m+1}^2 = B_1(b_{2m} + c_{2m}) + B_2(b_m^2 + c_m^2). \tag{2.11}$$

Applying the inequalities given by (1.6) in (2.11) for the coefficients c_m, c_{2m}, b_m and b_{2m}, we have

$$|a_{m+1}| \leq \sqrt{\frac{2B_1 + 2|B_2|}{(2m+1)(m+1)}}. \tag{2.12}$$

Substituting (2.8) and (2.9) into (2.11), we get

$$b_m^2 = \frac{(1+m)B_1(b_{2m} + c_{2m})}{(2m+1)B_1^2 - 2(m+1)B_2}. \tag{2.13}$$

From (2.8), (2.9) and (2.13), we get

$$(m+1)[(2m+1)B_1^2 - 2(m+1)B_2]a_{m+1}^2 = B_1^3(b_{2m} + c_{2m}). \tag{2.14}$$

Further, the equations (2.8) and (2.14) together with the equation (1.6) yield

$$|(m+1)[(2m+1)B_1^2 - 2(m+1)B_2]a_{m+1}^2| \leq 2B_1^3(1 - |b_m|^2). \tag{2.15}$$

From (2.4) and (2.15), we obtain

$$|a_{m+1}| \leq \frac{B_1\sqrt{2B_1}}{\sqrt{(m+1)[2(m+1)B_1 + |(2m+1)B_1^2 - 2(m+1)B_2|]}}. \tag{2.16}$$

Now, from (2.10), (2.12) and (2.16), we get

$$|a_{m+1}| \leq \min\left\{\frac{B_1}{m+1}, \sqrt{\frac{2B_1 + 2|B_2|}{(2m+1)(m+1)}}, \frac{B_1\sqrt{2B_1}}{\sqrt{(m+1)[2(m+1)B_1 + |(2m+1)B_1^2 - 2(m+1)B_2|]}}\right\}.$$

Next, in order to find the bound on $|a_{2m+1}|$, by substituting (2.7) from (2.5), we get

$$a_{2m+1} = \frac{m+1}{2}a_{m+1}^2 + \frac{B_1}{2(2m+1)}(b_{2m} - c_{2m}). \tag{2.17}$$

Then, in view of (2.4), (2.8) and (2.9), applying the inequalities in (1.6) for the coefficients b_{2m} and c_{2m}, we get

$$\begin{aligned} |a_{2m+1}| &\leq \frac{m+1}{2}|a_{m+1}|^2 + \frac{B_1}{2(2m+1)}(|b_{2m}| + |c_{2m}|) \\ &\leq \frac{m+1}{2}|a_{m+1}|^2 + \frac{B_1}{2m+1}(1 - |b_m|^2) \end{aligned}$$

$$\leq \left(\frac{m+1}{2}-\frac{(m+1)^2}{(2m+1)B_1}\right)|a_{m+1}|^2+\frac{B_1}{2m+1}. \tag{2.18}$$

From (2.10), (2.12), (2.16) and (2.18), we have the assertion (2.2). This completes the proof of Theorem 2.1.

Remark 2.1 The estimates of the coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ of Theorem 2.1 is the improvement of the estimates obtained in Theorem 1 of [15].

Setting $\varphi(z)=\left(\frac{1+z}{1-z}\right)^{\alpha}$ $(0<\alpha\leq 1)$ in Theorem 2.1, we have the following corollary.

Corollary 2.1 *Let $f(z)$ given by (1.3) be in the class $\mathcal{H}_{\Sigma,m}\left(\left(\frac{1+z}{1-z}\right)^{\alpha}\right)=\mathcal{H}^{\alpha}_{\Sigma_m}$. Then*

$$|a_{m+1}|\leq \min\left\{\frac{2\alpha}{m+1}, \sqrt{\frac{4\alpha+4\alpha^2}{(2m+1)(m+1)}}, \frac{2\alpha}{\sqrt{(m+1)(m+1+m\alpha)}}\right\},$$

$$|a_{2m+1}|\leq \begin{cases} \dfrac{2\alpha}{2m+1}, & 0<\alpha<\dfrac{m+1}{2m+1}; \\ \dfrac{6m\alpha^2+2\alpha^2}{(2m+1)(m+1+m\alpha)}, & \dfrac{m+1}{2m+1}\leq\alpha\leq 1. \end{cases}$$

Remark 2.2 The estimates of the coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ of Corollary 2.1 is the improvement of the estimates obtained in Theorem 2 of [19].

Setting $\varphi(z)=\frac{1+(1-2\beta)z}{1-z}$ $(0\leq\beta<1)$ in Theorem 2.1, we have the following corollary.

Corollary 2.2 *Let $f(z)$ given by (1.3) be in the class $\mathcal{H}_{\Sigma,m}\left(\frac{1+(1-2\beta)z}{1-z}\right)=\mathcal{H}^{\beta}_{\Sigma_m}$. Then*

$$|a_{m+1}|\leq \min\left\{\frac{2(1-\beta)}{m+1}, \sqrt{\frac{8(1-\beta)}{(2m+1)(m+1)}}, \frac{2(1-\beta)}{\sqrt{(m+1)[m+1+|m-\beta(2m+1)|]}}\right\},$$

$$|a_{2m+1}|\leq \begin{cases} \dfrac{2(1-\beta)}{2m+1}, & \dfrac{m}{2m+1}<\beta<1; \\ \dfrac{4(1-\beta)}{2m+1}-\dfrac{2(m+1)}{(2m+1)^2}, & 0\leq\beta\leq\dfrac{m}{2m+1}. \end{cases}$$

Remark 2.3 The estimates of the coefficients $|a_{m+1}|$ and $|a_{2m+1}|$ of Corollary 2.2 is the improvement of the estimates obtained in Theorem 3 of [19].

Setting $m=1$ in Theorem 2.1, we have the following corollary.

Corollary 2.3 *Let $f(z)$ given by (1.3) be in the class $\mathcal{H}_{\Sigma,1}(\varphi)=\mathcal{H}_{\Sigma}(\varphi)$. Then*

$$|a_2|\leq\min\left\{\frac{B_1}{2}, \sqrt{\frac{B_1+|B_2|}{3}}, \frac{B_1\sqrt{B_1}}{\sqrt{4B_1+|3B_1^2-4B_2|}}\right\},$$

$$|a_3| \leq \begin{cases} \dfrac{B_1}{3}, & B_1 < \dfrac{4}{3}; \\ \min\left\{\dfrac{B_1^2}{4}, \left(1 - \dfrac{4}{3B_1}\right)\dfrac{B_1 + |B_2|}{3} + \dfrac{B_1}{3}, \right. \\ \qquad \left. \left(1 - \dfrac{4}{3B_1}\right)\dfrac{B_1^3}{4B_1 + |3B_1^2 - 4B_2|} + \dfrac{B_1}{3}\right\}, & B_1 \geq \dfrac{4}{3}. \end{cases}$$

Remark 2.4 The estimates of the coefficients $|a_2|$ and $|a_3|$ of Corollary 2.3 is the improvement of the estimates obtained in Theorem 2.1 of [13].

Setting $m = 1$ and $\varphi(z) = \left(\dfrac{1+z}{1-z}\right)^\gamma$ $(0 < \gamma \leq 1)$ in Theorem 2.1, we have the following corollary.

Corollary 2.4 *Let $f(z)$ given by* (1.3) *be in the class* $\mathcal{H}_{\Sigma,1}\left(\left(\dfrac{1+z}{1-z}\right)^\gamma\right)$. *Then*

$$|a_2| \leq \frac{\sqrt{2}\gamma}{\sqrt{2+\gamma}},$$

$$|a_3| \leq \begin{cases} \dfrac{2\gamma}{3}, & 0 < \gamma < \dfrac{2}{3}; \\ \dfrac{8\gamma^2}{6+3\gamma}, & \dfrac{2}{3} \leq \gamma \leq 1. \end{cases}$$

Remark 2.5 The estimates for $|a_3|$ asserted by Corollary 2.4 are more accurate than those given by Theorem 1 in Srivastava *et al.*[14].

Setting $m = 1$ and $\varphi(z) = \dfrac{1+(1-2\gamma)z}{1-z}$ $(0 \leq \gamma < 1)$ in Theorem 2.1, we have the following corollary.

Corollary 2.5 *Let $f(z)$ given by* (1.3) *be in the class* $\mathcal{H}_{\Sigma,1}\left(\dfrac{1+(1-2\gamma)z}{1-z}\right)$. *Then*

$$|a_2| \leq \frac{\sqrt{2}(1-\gamma)}{\sqrt{2+|1-3\gamma|}},$$

$$|a_3| \leq \begin{cases} \dfrac{2(1-\gamma)}{3}, & \dfrac{1}{3} < \gamma < 1; \\ \dfrac{8-12\gamma}{9}, & 0 \leq \gamma \leq \dfrac{1}{3}. \end{cases}$$

Remark 2.6 The estimates for $|a_2|$ and $|a_3|$ asserted by Corollary 2.5 are more accurate than those given by Theorem 2 in Srivastava *et al.*[14].

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