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Distributed Finite-Time Nash Equilibrium Seeking for Non-Cooperative Games

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Abstract. The paper aims to design a distributed algorithm for players in games such that the players can learn Nash equilibriums of non-cooperative games in finite time. We first consider the quadratic non-cooperative games and design estimate protocols for the players such that they can estimate all the other players' actions in distributed manners. In order to make the players track all the other players' real actions in finite time, a bounded gradient dynamics is designed for players to update their actions by using the estimate information. Then the algorithm is extended to more general non-cooperative games and it is proved that players' estimates can converge to all the other players' real actions in finite time and all players can learn the unique Nash equilibrium in finite time under mild assumptions. Finally, simulation examples are provided to verify the validity of the proposed finite-time distributed Nash equilibrium seeking algorithms.

AMS subject classifications: 91A43, 93C10, 93D99

Key words: Nash equilibrium, non-cooperative game, distributed algorithm, finite-time convergence.

1 Introduction

Non-cooperative game is an important branch of game in which the cost function of each player relies on its action and other players' actions and each player wants to minimize its cost function selfishly. Nash equilibrium seeking problem as one of research emphases of non-cooperative games, aims to find a strategy combination under which no player can minimize its cost function by changing its strategy individually [1].

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In recent years, various distributed Nash equilibrium seeking algorithms have been proposed under the problem setting where the players only know part of the decision information (see [2–4] and references therein). The players have limited decision information in networked games while their cost functions depend on global decision information concluding all players' strategies. For non-cooperative games, global decision information is the prerequisite for each player to make the best response to other players' strategies. In the existing literature for non-cooperative games with partial-decision information setting, a common idea is to use the estimation of global decision information to update players' strategies. To be more specific, the networked players exchange their estimates of all other players' strategies based on the underlying communication graph. Then, the consensus idea is used to make the estimated information converge to the real global decision information, and the gradient strategy is used to make the actions of players converge to Nash equilibrium.

The main difference between distributed optimization and non-cooperative game lies in that agents aim to minimize a sum of their local objective functions cooperatively [5,6] in distributed optimization while players want to minimize their own objective functions selfishly in non-cooperative games, i.e., the purpose of distributed optimization is to solve

$$\min_x \sum_{i=1}^N f_i(x),$$

while the purpose of players in non-cooperative games is

$$\min_{x_i} f_i(x), \quad \forall i=1,\cdots,N.$$

At present, some literatures have studied the problem of finite-time optimization (e.g., [7–9] and references therein). Although some authors have studied the accelerated Nash equilibrium seeking algorithm, for example, [10] proposed an accelerated version of the gradient play algorithm with a faster convergence for Nash equilibrium seeking problem and [11] investigated Nash equilibrium seeking problem via alternating direction method of multipliers, these works still stay on the results of asymptotic convergence.

Motivated by finite-time distributed optimization, in this paper we aim to solve the finite-time Nash equilibrium seeking problem. In this paper, the players still keep estimates on all the other players' actions and update their actions by employing these estimates. Gradient play is still adopted here. Firstly, in order to make the estimates track players' real actions in finite time, we make some improvements on the basis of gradient play to ensure the update rate of players' actions is bounded. Secondly, we combine sign function and the idea of consensus to design update algorithm for players' estimates. The analysis of finite-time Nash equilibrium seeking is carried out in two stages: (1) the estimates converge to players' actions in limited time under the assumption of connected communication graph; (2) all players' actions converge to Nash equilibrium in finite time after the first stage under some assumptions.