CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.2021.nla.06

Vol. **2**, No. 2, pp. 357-375
June 2021

A Receptance-Based Optimization Approach for Minimum Norm and Robust Partial Quadratic Eigenvalue Assignment

Min Lu[1] and Zheng-Jian Bai[2,*]

[1] *School of Mathematical Sciences, Zhejiang University, Hangzhou 310027, P.R. China.*

[2] *School of Mathematical Sciences and Fujian Provincial Key Laboratory on Mathematical Modeling & High Performance Scientific Computing, Xiamen University, Xiamen 361005, P.R. China.*

Received 26 June 2020; Accepted 26 January 2021

Abstract. This paper is concerned with finding a minimum norm and robust solution to the partial quadratic eigenvalue assignment problem for vibrating structures by active feedback control. We present a receptance-based optimization approach for solving this problem. We provide a new cost function to measure the robustness and the feedback norms simultaneously, where the robustness is measured by the unitarity or orthogonalization of the closed-loop eigenvector matrix. Based on the measured receptances, the system matrices and a few undesired open-loop eigenvalues and associated eigenvectors, we derive the explicit gradient expression of the cost function. Finally, we report some numerical results to show the effectiveness of our method.

AMS subject classifications: 65F18, 93B55, 93C15

Key words: Partial quadratic eigenvalue assignment, robustness, optimization method, receptance measurements.

1 Introduction

The active vibration control is often needed in many vibrating structures in structural engineering, including structural dynamics [14, 16, 30], earthquake engineering control [13], damped-gyroscopic system control [17], large flexible space structure control theory [6, 7, 19, 20], control of mechanical descriptor systems [21]. In practice, by using the

*Corresponding author. *Email addresses:* `lumin@vip.126.com` (M. Lu), `zjbai@xmu.edu.cn` (Z.-J. Bai)

 ©2021 Global-Science Press

finite element technique and feedback vibration control, a vibrating structure is often discretized as a second-order feedback control system as follows [16]:

$$M\ddot{\mathbf{x}}(t)+C\dot{\mathbf{x}}(t)+K\mathbf{x}(t)=B\mathbf{u}(t), \quad \mathbf{u}(t)=F^T\dot{\mathbf{x}}(t)+G^T\mathbf{x}(t), \tag{1.1}$$

where $M,C,K\in\mathbb{R}^{n\times n}$ stand for the mass, damping and stiffness matrices accordingly, t means time, $\mathbf{x}(t),\dot{\mathbf{x}}(t),\ddot{\mathbf{x}}(t)$ denote the displacement, velocity and acceleration vectors accordingly, $B\in\mathbb{R}^{n\times m}$ is a control matrix ($m\le n$), and $\mathbf{u}(t)\in\mathbb{C}^m$ is the control vector with the unknown feedback matrices $F,G\in\mathbb{R}^{n\times m}$. In many engineering applications, M, C and K are all real symmetric with M being positive definite and K being positive semi-definite [16].

The dynamics of a vibrating structure as (1.1) is represented as the natural frequencies and mode shapes. In fact, by the separation of variables, $\mathbf{x}(t)=\mathbf{x}e^{\lambda t}$, where $\mathbf{x}$ is a constant vector, the general solution to the homogeneous system of (1.1) is determined by the quadratic eigenvalue problem: $P(\lambda)\mathbf{x}\equiv(\lambda^2M+\lambda C+K)\mathbf{x}=0$, where $P(\lambda)$ is called the open-loop quadratic matrix pencil and λ is called an eigenvalue of $P(\lambda)$ with associated right eigenvector $\mathbf{x}\neq\mathbf{0}$. If M is nonsingular, then $P(\lambda)$ has $2n$ finite right eigenpairs $\{(\lambda_j,\mathbf{x}_j)\}_{j=1}^{2n}$ [35]. From (1.1) we have the following closed-loop quadratic matrix pencil:

$$P_c(\lambda)\equiv\lambda^2M+\lambda(C-BF^T)+(K-BG^T).$$

In this paper, we consider the following partial quadratic eigenvalue assignment problem (PQEAP) for the second-order control system (1.1): find the feedback matrices $F,G\in\mathbb{R}^{n\times m}$ such that the closed-loop pencil $P_c(\lambda)$ has the assigned eigenvalues $\{\mu_j\}_{j=1}^{p}$, which replace the undesired open-loop eigenvalues $\{\lambda_j\}_{j=1}^{p}$ ($p\ll n$) while the remaining large number of open-loop right eigenpairs $\{(\lambda_j,\mathbf{x}_j)\}_{j=p+1}^{2n}$ are retained (i.e., the no spill-over property is preserved).

In addition, a robust and minimum norm solution to the PQEAP is expected so that the sensitivity of the closed-loop eigenvalues and the feedback norms are minimized simultaneously. The minimum norm solution may reduce the energy consumption and noise influence while the robust solution reduce the sensitivity to the vibrating structure perturbation and thus improve the reliability.

The classical methods for solving the PQEAP and the robust and minimum norm PQEAP include constructive methods and optimization method based on the solution of Sylvester equation. Recently, some receptance-based methods have been proposed for the pole assignment problem and the PQEAP (see for instance [4, 22–24, 27–29, 32, 33, 37]). The robust and minimum norm PQEAP is also considered based on the measured receptances and system matrices [1, 3, 5, 34, 37].

In this paper, we propose a new receptance-based optimization approach for solving the robust and minimum norm PQEAP. This is motivated by [8, 10] and [12]. In [12], an optimization method was presented for the robust pole assignment for the first-order model such that the closed-loop eigenvector matrix is as orthogonalized as possible.