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Radial Transonic Shock Solutions to Euler-Poisson System with Varying Background Charge in an Annulus

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Abstract. This paper concerns both the structural and dynamical stabilities of radially symmetric transonic shock solutions for two-dimensional Euler-Poisson system in an annulus. The density of fixed, positively charged background ions is allowed to be different constants in supersonic and subsonic regimes. First, the existence and structural stability of a steady transonic shock solution are obtained by the monotonicity between the shock location and the density on the outer circle. Second, any radially symmetric transonic shock solution with respect to small perturbations of the initial data is shown to be dynamically stable. The proof relies on the decay estimates and coupled effects from electric field and geometry of the annulus, together with the methods from [18]. These results generalize previous stability results on transonic shock solutions for constant background charge.

AMS subject classifications: 35B35, 35L65, 35L67, 35Q81, 76H05

Key words: Euler-Poisson equations, radial symmetry, transonic shock, varying background charge, stability.

1 Introduction and main results

The propagation of electrons in submicron semiconductor devices and plasma is governed by the Euler-Poisson equations [22]. In this paper, we focus on the two-dimensional Euler-Poisson equations in an annulus

$$\Omega=\left\{(x,y):0<r_1<r=\sqrt{x^2+y^2}<r_2<+\infty\right\}$$

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as follows:

$$\begin{cases} \rho_t+(\rho u_1)_x+(\rho u_2)_y=0, \\ (\rho u_1)_t+(\rho u_1^2)_x+(\rho u_1u_2)_y+p_x=\rho E_1, \\ (\rho u_2)_t+(\rho u_1u_2)_x+(\rho u_2^2)_y+p_y=\rho E_2, \\ (E_1)_x+(E_2)_y=\rho-b, \end{cases} \tag{1.1}$$

where $\boldsymbol{u}=(u_1,u_2)$ and $\rho>0$ represent the macroscopic particle velocity field and electron density, respectively. $\boldsymbol{E}=(E_1,E_2)$ is the electric field generated by the Coulomb force of particles, and $b>0$ stands for the density of fixed, positively charged background ions. The pressure p is given by $p=A\rho^\gamma(A>0,\gamma>1)$ and thus satisfies

$$p(0)=0, \quad p'>0, \quad p''>0 \quad \text{for} \quad \rho>0, \quad p(+\infty)=+\infty.$$

Moreover, the local sound speed c and Mach number M are defined by $c(\rho)=\sqrt{p'(\rho)}$ and $M=|\boldsymbol{u}|/c$, respectively. The flow is called supersonic if $|\boldsymbol{u}|>c(\rho)$; subsonic if $|\boldsymbol{u}|<c(\rho)$; sonic if $|\boldsymbol{u}|=c(\rho)$. The pure supersonic and subsonic flows have been studied by many people (see [2,3,5,11,21,30] and references therein). On the other hand, the discontinuous transonic flow, that is, transonic shock contains a free boundary (shock) on the left of the subsonic region. This leads to some essential difficulties for mathematical analysis of transonic shock solutions.

For Euler system, Courant and Friedrichs [10] described the transonic shock phenomena in a de Laval nozzle. According to the above phenomena, there are numerous significant results about the existence and stability of steady transonic shock solutions to Euler system in a nozzle (see [7–9, 14, 17, 29, 31, 32] and references therein). The global in time stability of transonic shock solutions was investigated by Liu [16] and Xin and Yin [32]. In [16], the author used a wave front tracking variant of Glimm's scheme to prove that, for quasi-one-dimensional system, a weak transonic shock solution is dynamically stable in divergent nozzle and dynamically unstable in convergent nozzle. These results were improved by Rauch *et al.* [27]. Other related results about the transonic flows can be found [6, 28, 33].

Concerning Euler-Poisson system, there have been only a few results for the transonic shock solutions. In one-dimensional case, a transonic shock problem with a linear pressure $p(\rho)=k\rho$ and special boundary conditions was discussed in [1]. For more general case, Gamba [13] constructed a transonic shock solution, which may contain boundary layers due to the technical limit. A thorough study of the transonic shock solutions for one-dimensional Euler-Poisson equations with a constant background charge $b=b_0$ in flat nozzles was given by Luo and Xin [19], where the existence, non-existence, uniqueness and non-uniqueness of solutions with transonic shock were established. Bae and Park [4] established the well-posedness of radial transonic shock problem for Euler-Poisson equations in a two-dimensional convergent nozzle under a strong effect of self-generated electric field. Luo *et al.* [18] proved that a steady transonic shock solution with supersonic background charge, obtained in [19], is structurally stable under small perturbations of the background charge and is dynamically stable with respect to small perturbation of