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An Approach to Obtain the Correct Shock Speed for Euler Equations with Stiff Detonation

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Abstract. Incorrect propagation speed of discontinuities may occur by straightforward application of standard dissipative schemes for problems that contain stiff source terms for underresolved grids even for time steps within the CFL condition. By examining the dissipative discretized counterpart of the Euler equations for a detonation problem that consists of a single reaction, detailed analysis on the spurious wave pattern is presented employing the fractional step method, which utilizes the Strang splitting. With the help of physical arguments, a threshold values method (TVM), which can be extended to more complicated stiff problems, is developed to eliminate the wrong shock speed phenomena. Several single reaction detonations as well as multi-species and multi-reaction detonation test cases with strong stiffness are examined to illustrate the performance of the TVM approach.

AMS subject classifications: 35Q31, 76L05

Key words: Detonation, spurious behavior, reactive Euler equations, threshold values method, stiffness.

1 Introduction

In simulating the reactive Euler equations with the homogeneous source terms, often applied in field of combustion and high speed chemical reacting, a well-known spurious numerical phenomenon which was observed firstly by Collela et al. [1], may occur if the equations are solved in the under-resolved conditions, namely the coarse grid, large time step or other combinations in conjunction with the type of spatial scheme and type of temporal discretization etc. [2–4]. By properly defining a model problem with a stiff source term, LeVeque and Yee [5] reveal that the typical spurious behavior which is the propagation error of the detonation wave, is chiefly due to the numerical dissipation

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contained in the schemes, which smears the discontinuity fronts and activates the source terms in a non-physical manner. Since then, this topic has attracted a great deal of attention.

Lafon and Yee [3,4] indicated that the spurious steady state of nonlinear source terms can be linked to the wrong shock speed by getting trapped at one of the stationary solutions, depending on the combination of numerical method, initial data, time step and grid spacing. Griffiths et al. [2] analyzed the different methods of numerically treating the stiff source and their accompanied spurious wave propagation phenomena. Yee et al. [6,7] followed to investigate the role of CFL playing in the spurious behavior and found the counter-intuitive behavior, which leads to the conclusion that the traditional concept of CFL guideline needs to be revised when extending to the reactive Euler equation systems. Recently, Zhang and Wang [8] give a reasonable explanation that the oscillation of the parameter of an intermediate state which is a decisive factor to decide whether or not the spurious solution will happen, is the likely cause of the counter-intuitive behavior.

Many other researchers focus on designing the new schemes or models to avoid this spurious numerical solution in the under-resolved computational conditions. During the last two decades, several innovative numerical methods, such as the level set and front tracking methods [9–13]; random choice method [14–16]; fractional step method [17]; random projection method (RPM) [18–20]; subcell resolution method [22,23]; MinMax Method [21]; equilibrium state method (ESM) [24] and many other works [25–31], have been proposed successively. A comprehensive review of the last two decades of this field can be obtained in [22]. In spite of being able to remove or delay the appearance of the spurious solution to some extent, these methods cannot be widely used due to some limitations. For example, existing methods are either confined to a particular flow type or restricted to certain stiffness of the reaction terms. When stiffness of the source term increases, some of the methods would break down even for a single reaction case.

The present work is a sequel to [8] to extend the idea to the Euler equations with stiff detonation. By examining the dissipative discretized counterpart of the Euler equations for a detonation problem consisting of a single reaction, a detail analysis on the spurious wave pattern is presented employing the fractional step method using the Strang splitting. Additionally, a novel method called the threshold values method (TVM for short) is proposed as a modification to the fractional step method with the help of physical arguments. Several single reaction detonation as well as multi-species and multi-reaction detonation test cases with strong stiffness are examined to illustrate the performance of the TVM approach.

2 The standard numerical method for the reactive Euler equations

The governing equations are usually used to simulate the inviscid, one-dimensional propagation of a detonation wave, representing conservation of mass, momentum, energy and