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Finite Element Analysis for Nonstationary Magneto-Heat Coupling Problem

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Abstract. This paper is devoted to finite element analysis for the magneto-heat coupling model which governs the electromagnetic fields in large power transformers. The model, which couples Maxwell's equations and Heat equation through Ohmic heat source, is nonlinear. First we derive an equivalent weak formulation for the nonlinear magneto-heat model. We propose a linearized and temporally discrete scheme to approximate the continuous problem. The well-posedness and error estimates are proven for the semi-discrete scheme. Based on the results, we propose a fully discrete finite element problem and prove the error estimates for the approximate solutions. To validate the magneto-heat model and verify the efficiency of the finite element method, we compute an engineering benchmark problem of the International Compumag Society, P21^b-MN. The numerical results agree well with experimental data.

AMS subject classifications: 65N15, 65N30, 78A25

Key words: Magneto-heat coupling model, eddy current problem, Maxwell equations, finite element method.

1 Introduction

In numerical simulation of power transforms, eddy current loss accounts for the major part of the total energy loss. The Ohmic heat and magnetic hysteresis lead to energy loss and can damage the devices of a power transformer (see [8, 9]). In [18], the authors proposed a magnetic-heat coupling model for large power transformers and established

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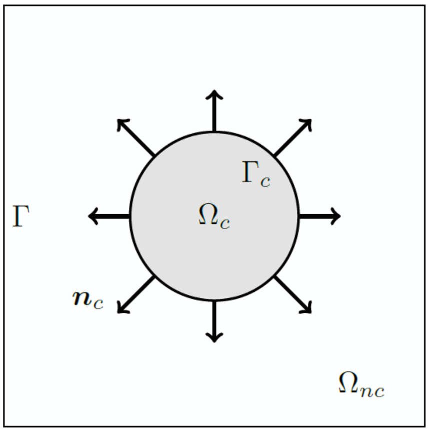

Figure 1: A 2D illustration of the problem geometry, $\bar{\Omega}=\bar{\Omega}_c\cup\bar{\Omega}_{nc}$.

the well-posedness of the problem. The model consists of Maxwell's equations and the heat equation with radiation condition on the boundary of conductors. It describes how the energy is transferred from electromagnetic fields to Ohmic heat, and the authors proposed a weak formulation for the model and established the well-posedness of the problem. This paper is a subsequent work of [18] and is focused on numerical analysis for the magneto-heat coupling model.

We study the magneto-heat coupling problem. Let $\Omega\subset\mathbb{R}^3$ be a truncation domain which contains all inhomogeneities like conductors, coils etc. The material outside of Ω is the air which is homogeneous and insulating. Without loss of generality, we assume Ω is a cube. Suppose $\bar{\Omega}=\bar{\Omega}_c\cup\bar{\Omega}_{nc}$ where Ω_c is the conducting region and Ω_{nc} the insulating region. Let $\Gamma=\partial\Omega$ denote the boundary of Ω and $\Gamma_c=\partial\Omega_c$ the boundary of Ω_c. Throughout the paper, we assume $\bar{\Omega}_c\subset\Omega$, $\Gamma\cap\Gamma_c=\emptyset$, and that Ω_c is a Lipschitz domain. Let T be the final time. The electromagnetic fields of the power transformer are governed by the Maxwell's equations

$$\partial_t\boldsymbol{B}+\mathbf{curl}\,\boldsymbol{E}=0 \quad \text{in } \Omega\times(0,T), \tag{1.1a}$$
$$\mathbf{curl}\,\boldsymbol{H}-\sigma\boldsymbol{E}=\boldsymbol{f} \quad \text{in } \Omega\times(0,T), \tag{1.1b}$$
$$\boldsymbol{E}\times\boldsymbol{n}=0 \quad \text{on } \Gamma\times(0,T), \tag{1.1c}$$
$$\boldsymbol{E}(0)=0 \quad \text{in } \Omega, \tag{1.1d}$$

where $\boldsymbol{E}$ stands for the electric field, $\boldsymbol{B}$ the magnetic flux density, $\boldsymbol{H}$ the magnetic field, σ the conductivity, and $\partial_t=\frac{\partial}{\partial t}$ the partial derivative with respect to t. The temperature θ is governed by the heat equation with initial and boundary conditions

$$C_\rho\partial_t\theta-\nabla\cdot(\kappa\nabla\theta)=\sigma|\boldsymbol{E}|^2 \quad \text{in } \Omega_c\times(0,T), \tag{1.2a}$$
$$-\kappa\partial_{\boldsymbol{n}_c}\theta=\bar{q}+\lambda(\theta,\partial_{\boldsymbol{n}_c}\theta)\cdot(\theta-\theta_0) \quad \text{on } \Gamma_c, \tag{1.2b}$$
$$\theta(0)=\theta_0 \quad \text{in } \Omega_c, \tag{1.2c}$$