

Parametric POD-Galerkin Model Order Reduction for Unsteady-State Heat Transfer Problems

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Abstract. A parametric reduced order model based on proper orthogonal decomposition with Galerkin projection has been developed and applied for the modeling of heat transport in T-junction pipes which are widely found in nuclear power reactor cooling systems. Thermal mixing of different temperature coolants in T-junction pipes leads to temperature fluctuations and this could potentially cause thermal fatigue in the pipe walls. The novelty of this paper is the development of a parametric ROM considering the three dimensional, incompressible, unsteady Navier-Stokes equations coupled with the heat transport equation in a finite volume regime. Two different parametric cases are presented in this paper: parametrization of the inlet temperatures and parametrization of the kinematic viscosity. Different training spaces are considered and the results are compared against the full order model. The first test case results to a computational speed-up factor of 374 while the second test case to one of 211.

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Key words: Proper orthogonal decomposition, finite volume approximation, Poisson equation for pressure, inf-sup approximation, supremizer velocity space enrichment, Navier-Stokes equations.

1 Introduction

Partial differential equations (PDEs) describe a variety of physical systems occurring in nature and in engineering. PDEs are complex and generally nonlinear and their numerical solution requires considerable computational effort. For example, fluid flow,

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a phenomenon very common in many engineering fields, is governed by the Navier-Stokes equations and accurate numerical solutions provide vital insight into complex physical processes. Analytical solution of these equations is impossible in almost all circumstances. For this reason, computational fluid dynamics (CFD) has seen progressive development since the 1970s and is now capable of solving many practical problems in fluid flow and heat transfer. With the continued development of improved algorithms and increasing computational power, CFD is now used in various engineering fields such as aerospace, nuclear, civil, mechanical as well as non-engineering fields such as neuroscience and meteorology etc.

Despite its popularity and applicability, the computational burden for simulating realistic large scale and many query systems is still very high, even with the use of supercomputers. A good example of the challenges involved can be found in nuclear applications, where turbulence, multiphase flow and heat transfer phenomena occur in complex geometries; a fairly accurate CFD simulation of a single instance of an accident case scenario could take months or more to be performed. To address these challenges, Systems Codes (SC), such as RELAP, CATHARE, etc and sub-channel codes (COBRA, etc), constitute phenomenological reduced order methods based on considerable limiting physical assumptions. These codes, that were developed in the 1950s, rely on major physical and geometrical simplifications, such as averaging over the flow cross section leading to essentially 1D simulations. These simplifications can save great amounts of computational time. However, the compromise is that they rely exclusively on experimental and phenomenological correlations to take account of heat transfer and turbulence and the like. In particular, these assumptions are particularly inadequate for 3D flows. In the recent years although these codes have been improved allowing some limited 3D capability, the accuracy is still inadequate and their application is very limited. The same applies in the field of neutronics for the study of reactor dynamics. Geometrical and physical simplifications are made to the governing equations in order to obtain a computationally affordable model. These simplifications include 1D geometries, homogenous core dynamics, uniform axial fluxes, etc. The challenge then, is to bridge the considerable gap between high fidelity full-order models (eg CFD and its variants) and these oversimplistic reduced order models (systems and sub-channel codes).

Modern reduced order models (ROMs) [1–3] have been proposed as an alternative way of approximating full-order systems (such as those arising in conventional CFD) in a more sophisticated and reliable way. Unlike phenomenological methods, modern ROMs potentially retain the high fidelity of the full order model (FOM) while exhibiting performance akin to phenomenological methods. Reduced order modeling is a highly promising area, which is currently flourishing in the science and engineering community.

An essential tool in the development of ROMs is the Proper orthogonal decomposition (POD) or Karhunen-Loève decomposition. Originally conceived as a data analysis method for finding an optimal lower-dimensional orthonormal basis in a least-squares sense, POD can be used as a model order reduction method for multidimensional dynamical systems, using data from high fidelity simulations (in this case CFD) or from ex-