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A Novel Full-Euler Low Mach Number IMEX Splitting

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Abstract. In this paper, we introduce an extension of a splitting method for singularly perturbed equations, the so-called RS-IMEX splitting [Kaiser et al., Journal of Scientific Computing, 70(3), 1390–1407], to deal with the fully compressible Euler equations. The straightforward application of the splitting yields sub-equations that are, due to the occurrence of complex eigenvalues, not hyperbolic. A modification, slightly changing the convective flux, is introduced that overcomes this issue. It is shown that the splitting gives rise to a discretization that respects the low-Mach number limit of the Euler equations; numerical results using finite volume and discontinuous Galerkin schemes show the potential of the discretization.

AMS subject classifications: 35L81, 65M08, 65M60, 76M45

Key words: Euler equations, low-Mach, IMEX Runge-Kutta, RS-IMEX.

1 Introduction

The modeling of many processes in fluid dynamics requires the solution of the compressible Euler equations. This hyperbolic set of equations contains propagation waves at different speeds. At low Mach numbers, the propagation speeds of the waves differ by

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several orders of magnitude leading to a stiff behavior of the equation system. Classical explicit time integration schemes would require the resolution of all waves for stability reasons, even though the contribution of the fast waves on the solution can be negligible for many applications. Fully implicit schemes can overcome this stability issue, but they require specially designed numerical fluxes and need the solution of large nonlinear systems, see e.g. [3,35]. One way to obtain a stable and efficient numerical method is to split the equations into a stiff and a non-stiff part and handle those parts implicitly and explicitly, respectively. Such a procedure leads to IMEX schemes, see, e.g., [2,6,18] and the references therein. This approach is particularly advantageous in cases where the time resolved resolution of the slow waves is of interest. The choice of the splitting is important. It determines such crucial properties as stability, accuracy and efficiency. It is therefore not surprising that many people have worked on identifying suitable splittings for various types of the Euler equations, starting from the groundbreaking work of Klein [26]. To name a few – this list is by no means exhaustive – we refer to [4,8–11,13,15,22,29,34]. The present splittings have different drawbacks such as the need for solving an elliptic equation or large nonlinear systems, requiring staggered meshes, being limited to low order discretization or not being able to be applied to the full Euler equations. Recently, a new splitting for the isentropic Euler equations based on the solution of the incompressible Euler equations has been introduced, see [21–23, 36, 37]. This splitting can be combined with a high order discretization in space and time and remains linear in the implicit part which significantly simplifies the solution of the associated system. Applying it in a straightforward manner to the full Euler equations of gas dynamics results in a scheme where the explicit part is no longer hyperbolic. This is of course an unwanted feature, as the method can become instable, which will be discussed in this work.

The purpose of this paper is to propose a modification of the splitting to overcome this issue, thereby obtaining a stable and accurate scheme for the Euler equations at low Mach numbers. The final splitting is combined with an IMEX Runge-Kutta method and is shown to be asymptotically consistent. Then, spatial discretization is achieved via either a finite volume [33] approach or a discontinuous Galerkin approach [17, 27]. In future it would be interesting to investigate how our methodology can be generalized for further non-standard space approximations of the Euler equations known in the literature, such as the ALE-multi-moment finite volume scheme [19] or high-resolution Lagrangian methods [30].

The paper is structured as follows: In Section 2 we introduce the RS-IMEX splitting for the Euler equations and describe how it can be modified in order to obtain a hyperbolic explicit and implicit system. Following, Section 3 introduces the numerical discretization of the novel splitting. Here, the asymptotic consistency of the semi discrete scheme is proven in Section 3.2 and the spatial discretization is introduced in Section 3.3. The limitations of the splitting are discussed in the subsequent section (Section 4). Following, the low Mach number capabilities of the scheme are illustrated with suitable testcases in Section 5. Finally, the paper is closed with a conclusion and outlook (Section 6).