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Long Wave Interaction with a Partially Immersed Body. Part I: Mathematical Models

Gayaz Khakimzyanov[1] and Denys Dutykh[2,3,*]

[1] *Institute of Computational Technologies of Siberian Branch of the Russian Academy of Sciences, Novosibirsk 630090, Russia.*
[2] *Univ. Grenoble Alpes, Univ. Savoie Mont Blanc, CNRS, LAMA, 73000 Chambéry, France.*
[3] *LAMA, UMR 5127 CNRS, Université Savoie Mont Blanc, Campus Scientifique, 73376 Le Bourget-du-Lac Cedex, France.*

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Abstract. In the present article we consider the problem of wave interaction with a partially immersed, but floating body. We assume that the motion of the body is prescribed. The general mathematical formulation for this problem is presented in the framework of a hierarchy of mathematical models. Namely, in this first part we formulate the problem at every hierarchical level. The special attention is paid to fully nonlinear and weakly dispersive models since they are most likely to be used in practice. For this model we have to consider separately the inner (under the body) and outer domains. Various approached to the gluing of solutions at the boundary is discussed as well. We propose several strategies which ensure the global conservation or continuity of some important physical quantities.

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1 Introduction

In the design process of various floating devices, one of the main parameters to take into account is the expected wave run-up magnitude during unavoidable over-topping events in rough seas. Clearly, there is a need to analyze large parameter spaces, *i.e.* wave/wind states, relative dimensions, orientations, *etc.* in order to obtain a nearly-optimal design.

*Corresponding author. *Email addresses:* `Khak@ict.nsc.ru` (G. Khakimzyanov), `Denys.Dutykh@ univ-smb.fr` (D. Dutykh)

In this way we arrive naturally to the need of development of fast and accurate numerical algorithms to simulate wave fields interaction with floating structures. The methods developed in coastal/naval engineering communities are based on an important number of simplifying assumptions (see *e.g.* [39, 72]) to obtain quick estimations of required parameters. There are also well-developed analytical methods restricted essentially to linear problems [63]. The goal of our study is to introduce into this topic a more nonlinear description along with efficient methods to solve equations numerically. As a result, we would like to be able to estimate even local characteristics of the flow. We shall make at some point two main simplifying assumptions:

1. The waves are long, *i.e. weakly* dispersive.

2. The object is floating, but fixed in horizontal directions† (in contrast to freely floating objects).

Because of the second assumption, we shall speak below about a *partially immersed body*. Nevertheless, the model predictions will be checked against spare experimental data [47]. Despite these simplifying assumptions, there are practically important situations, where they hold true. To give an example, the resonant wave pumping device analyzed in [12] falls perfectly in the framework presented in this study.

The well-studied topic is the wave generation by moving structures such as ships. Practical interest of such works is quite obvious. The first analytical steps in this direction were done by Lord Kelvin. We can refer also, for example, to early numerical (finite difference) attempts to compute the wave field behind a ship [70]. In the problem considered in our study the structure is fixed and we are interested in wave interaction with it. Moreover, we are looking at generated wave fields behind *and* in front of the floating object.

The topic of numerical modelling of the wave/(floating or immersed) body interaction attracted much attention in the recent years. In water wave theory the most studied situation both by analytical and numerical techniques is the wave interaction with a single [45, 102] or with an array [101] of floating/fixed circular cylinders. In [45] the cylinder was allowed to move in vertical direction. Most of the numerical studies are based on the boundary integral equations method, while analytical investigations focus mainly on linear or, exceptionally, weakly nonlinear formulations. However, there are a few exceptions. A mixed Eulerian-Lagrangian method was applied to describe wave-induced motions of a floating body in [48]. On the other hand, an Eulerian spectral element method was recently applied to wave/body interaction problems in [33]. The ultimate goal of such investigations is to propose a robust and efficient methodology for the simulation of floating real world objects such as boats [76] and/or wave energy converters [13,37,81]. In such complex applications sometimes even the full Navier-Stokes equations are solved using state-of-the-art computational techniques [62, 103]. For instance,

†In our modelling we allow the object to move freely in the vertical direction according to a prescribed law.