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Physical Valid Scale of General Continuum Models in Unsteady Flow

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Abstract. A variety of numerical simulations based on continuum medium models have been carried out in unsteady flows to explore the physical characteristics qualitatively. However, there still lack quantitative valid scales for these models, in which scale the solutions by simulation can be treated as physically credible. In this work, Euler and Navier-Stokes models are implemented on the typical unsteady flow with discontinuities: Sod shock tube problem. Firstly, based on the Boltzmann model whose valid scale is molecule kinetic scale, we compare the values of different moments of Euler and Navier-Stokes to Boltzmann with the evolution of flow. The results provide the valid scales of Euler and Navier-Stokes models quantitatively. Secondly, following the real conditions and parameters in real air, the physical characteristics of waves' generation and evolution in Sod shock tube are observed and analyzed numerically at the microscopic level for the first time. Our results present the reference scales for the choice of minimal simulation scale in which the continuum medium models are used, and for the criteria whether the flow characteristics obtained by simulations can be treated as real physical characteristics.

AMS subject classifications: 82B21, 76-XX
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1 Introduction

Physical results at a concerned scale can be obtained if both of the two following points are guaranteed: (1) the physical characteristics of real flows at the concerned scale are

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correctly modeled by the selected fluid dynamics model; (2) the corresponding modeli.e. governing equations, is solved by an adaptive numerical method with acceptable error. For the former, different models are established based on different assumptions, and there exists a corresponding minimal valid modeling scale $l_{\min}$, greater than which scale, the solutions obtained by the model can capture the physical characteristics. Although this fact has been widely accepted, the $l_{\min}$ of some generally used continuum medium models have not been quantitatively investigated, such as Euler and Navier-Stokes (NS) models.

The Boltzmann model is a probability statistical theory describing the motions of molecules by including both free transport and collision effect in molecule kinetic scale, i.e. scale of molecule mean free path. Euler [1] and NS [2] models are the most widely used continuous medium models which are established based on continuous medium and local thermodynamic equilibrium hypothesis. Holian [3] and Hoover [4] have found that the continuous medium models would fail nearby the shockwave in theory. For numerical study, the continuous medium models cannot capture the physical characteristics of nonequilibrium effect around the shock wave whose characteristic scale is at the level of mean free path [5]. The kinetic methods based on Boltzmann model have applied to researches of the nonequilibrium structure of shock wave [6–8]. The comparison of kinetic model and continuum model applied to the structure of shock wave has been studied by Kudryavtsev, Ivanov and Zeitoun [9, 10]. They drew a conclusion that Navier-Stokes model highly underestimate the stationary shockwave thickness, especially at high Mach numbers. However, with the limit of the current computing power, continuous medium models are still the main methods to simulate the macroscopic shock wave structure. Therefore, it is of great concern that from which scale the solutions of continuous medium models are physically credible. Although previous studies [11, 12] proposed some qualitatively applicable scale range of Euler and NS equations, there still lacks a quantitative investigation. For the typical Sod shock tube problem [13] is a representative flow involving the generation and propagation of waves and the capturing of different kinds of discontinuities, it is often used to test the performance of numerical scheme. In this work, it is chosen to measure the physical valid scale of different models quantitatively. As the Knudsen number in such situation is relatively large, continuous models are compared with kinetic model in different moments such as density, velocity, temperature, heat flux and shear stress.

2 Governing equations

In our numerical experiments, the Euler and NS equations are given as follows:

$$\partial_t \mathbf{U} + \partial_x (\mathbf{F} - \mathbf{F}_v) = 0, \tag{2.1a}$$

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho u \\ \rho E \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ (\rho E + p) u \end{pmatrix}, \quad \mathbf{F}_v = \begin{pmatrix} 0 \\ 4\mu u_x/3 \\ 4u\mu u_x/3 + \kappa \partial_x T \end{pmatrix}, \tag{2.1b}$$