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A Three-Level Multi-Continua Upscaling Method for Flow Problems in Fractured Porous Media

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Abstract. Traditional two level upscaling techniques suffer from a high offline cost when the coarse grid size is much larger than the fine grid size one. Thus, multilevel methods are desirable for problems with complex heterogeneities and high contrast. In this paper, we propose a novel three-level upscaling method for flow problems in fractured porous media. Our method starts with a fine grid discretization for the system involving fractured porous media. In the next step, based on the fine grid model, we construct a nonlocal multi-continua upscaling (NLMC) method using an intermediate grid. The system resulting from NLMC gives solutions that have physical meaning. In order to enhance locality, the grid size of the intermediate grid needs to be relatively small, and this motivates using such an intermediate grid. However, the resulting NLMC upscaled system has a relatively large dimension. This motivates a further step of dimension reduction. In particular, we will apply the idea of the Generalized Multiscale Finite Element Method (GMsFEM) to the NLMC system to obtain a final reduced model. We present simulation results for a two-dimensional model problem with a large number of fractures using the proposed three-level method.

AMS subject classifications: 65N08, 65N30, 65M22

Key words: Multiscale method, three-level scheme, multicontinuum, upscaling, GMsFEM, NLMC, fractured porous media.

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1 Introduction

A fast and accurate solution of flow problems in fractured porous media is an important component in reservoir simulation. Direct numerical simulation requires using a very fine grid that resolves all scales and heterogeneities. The resulting discrete formulation on the fine grid leads to a very large system of equations that is computationally expensive to solve. To reduce the dimension of the system, multiscale methods or upscaling techniques are necessary [14, 17, 20, 25, 26, 32]. We will, in this paper, focus on a class of multiscale methods based on local multiscale basis functions. In typical two level methods, multiscale basis functions are constructed locally, namely, within a coarse block or a union of several coarse blocks of an underlying coarse mesh, which does not necessarily resolve any scale. Constructing multiscale basis functions involves solutions, using the fine grid, of some local problems, which can be expensive for the case when coarse grid size is much larger than the fine grid size one [11]. Therefore, problems with very large disparate scales require some coarsening techniques or multilevel methods [27]. The commonly used techniques for such problems are the re-iterated homogenization methods or multilevel multiscale methods [3, 11, 24, 27–29, 34, 39]. In multilevel multiscale approaches, multiple levels of coarsening are constructed by a recursive application of the basic two level method with the aim of improving computational efficiency. The main advantage of multilevel methods is to avoid solving local problems of large dimensions.

In our previous works, we developed multiscale model reduction techniques based on the Generalized Multiscale Finite Element Method (GMsFEM) for flow in fractured porous media [1,2,7,18]. The general idea of GMsFEM is to design suitable spectral problems on some snapshot spaces to obtain dominant modes of the solutions. These dominant modes are used to construct the required multiscale basis functions [5, 6, 15, 16]. The resulting multiscale space contains basis functions that take into account the microscale heterogeneities as well as high contrast and channelized effects, and the resulting multiscale scale solution provides an accurate and efficient approximation of the fine scale solution. We remark that the GMsFEM is related to the Proper Orthogonal Decomposition (POD) (c.f. [16]) in the way that the GMsFEM constructs multiscale basis functions that optimize an appropriate error within a finite dimensional space. The error of the GMsFEM has a spectral decay and is inversely proportional to the eigenvalues of the spectral problems used for constructing basis functions.

Recently, the authors in [9, 12] proposed a new Constraint Energy Minimizing GMsFEM (CEM-GMsFEM) with the aim of finding a multiscale method with a coarse mesh dependent convergence. Constructing the multiscale space starts with an auxiliary space, which consists of eigenfunctions of a local spectral problem, and is defined for each coarse element. Using the auxiliary space, one can obtain the required multiscale basis functions by solving a constraint energy minimization problem. The resulting multiscale basis functions have an exponential decay away from the coarse element for which the basis functions are formulated. Therefore, the multiscale basis functions are only numerically computed in an oversampled region defined by enlarging the target coarse element