An Efficient DDG/FV Hybrid Method for
3D Viscous Flow Simulations on Tetrahedral Grids

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Abstract. A new hybrid reconstruction scheme DDG/FV is developed in this work by combining the DDG method and DG/FV hybrid scheme developed in the authors' previous work [1–4] to simulate three-dimensional compressible viscous flow on tetrahedral grids. The extended von Neumann stencils are used in the reconstruction process to ensure the linear stability, and the L_2 projection and the least-squares method are adopted to reconstruct higher order distributions for higher accuracy and robustness. In addition, a quadrature-free L_2 projection based on orthogonal basis functions is implemented to improve the efficiency of reconstruction. Three typical test cases, including the 3D Couette flow, laminar flows over an analytical 3D body of revolution and over a sphere, are simulated to validate the accuracy and efficiency of DDG/FV method. The numerical results demonstrate that the DDG scheme can accelerate the convergence history compared with widely-used BR2 scheme. More attractively, the new DDG/FV hybrid method can deliver the same accuracy as BR2-DG method with more than 2 times of efficiency improvement in solving 3D Navier-Stokes equations on tetrahedral grids, and even one-order of magnitude faster in some cases, which shows good potential in future realistic applications.

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Key words: Direct discontinuous Galerkin method, DG/FV hybrid method, high order method, unstructured grid, viscous flow.

1   Introduction

High order methods on unstructured/hybrid grids have become one of the most popular fields in Computational Fluid Dynamics (CFD) in recent years. The Discontinuous

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Galerkin (DG) method, as one of the representatives of high order methods on unstructured grids, combines many advantageous features of finite volume methods (FVM) and finite element methods (FEM): (1) excellent mathematical properties including conservation, stability, and convergence; (2) easy to extend to higher-order approximation by increasing the order of numerical polynomial solution; (3) good at dealing with complex geometries, physical boundaries and hanging nodes; (4) compactness and potential for large scale parallel computation; (5) *hp*-adaptive can be easily implemented via refining or coarsening the cells and setting different order of numerical polynomial solution in each cell. Based on these advantages, DG method has become the main topic of many international programs on high order numerical algorithms, such as ADIGMA [5] and IDHOM [6] in Europe.

This paper will focus on two aspects of the studies on DG method: one is the development of consistent, compact and efficient viscous term processing methods for Navier-Stokes equations; the other is to reduce the huge computational cost and memory requirement in DG method.

Many researchers have proposed a variety of methods to construct numerical flux for diffusion equation, since taking the average of the derivatives on both sides of an interface as the derivatives of primitive variables on this interface without considering possible jumps will lead to inconsistency [7]. The earliest method to address this issue is the interior penalty (IP) method, which was proposed and studied in Refs. [8–10]. The numerical flux in IP method includes not only the average of left and right states, but also the penalty term which takes the jump of solutions at cell interfaces into account. By improving IP method, Hartmann obtained the symmetric IP (SIP) method [11,12] and the new SIP method [13] afterwards. BR1 [14] and Local DG (LDG) [15] are another type of methods to deal with the diffusion flux. It is based on the idea to convert a second-order equation into two first-order equations by introducing an auxiliary variable, since DG method excels in solving first-order equations. The difference between BR1 method and LDG method is the choice of numerical flux at cell interfaces. Replacing the global lifting operator with the local lifting operator to enhance the compactness in the BR1 method, the BR2 scheme [16] was developed and has become one of the most widely used numerical viscous flux schemes in DG method. Following the similar idea, the compact DG (CDG) method [17] as an improvement of LDG method, assigns the lifting operator of the auxiliary equation to each cell interface for compactness in multidimensional problems. In 2005, the recover-based DG (RDG) [18,19] was proposed to recover a smooth continuous solution based on the recovery principle. Interested readers can refer to Refs. [7] and [20] for the comparative studies of several DG methods mentioned above.

In 2008, motivated by the trace formula of the solution derivative of the heat equation, Liu and Yan [21] proposed the direct discontinuous Galerkin (DDG) method with the numerical flux involving the average of left and right states and the jumps of even order derivatives of solution at cell interfaces. In 2010, Liu and Yan [22] further provided an improved version of DDG method, adding an interface correction term to avoid the higher-order derivative terms. The DDG method not only maintains the compactness