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Constraint Energy Minimizing Generalized Multiscale Finite Element Method for High-Contrast Linear Elasticity Problem

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Abstract. In this paper, we consider the offline and online Constraint Energy Minimizing Generalized Multiscale Finite Element Method (CEM-GMsFEM) for high-contrast linear elasticity problem. Offline basis construction starts with an auxiliary multiscale space by solving local spectral problems. We select eigenfunctions that correspond to a few small eigenvalues to form the auxiliary space. Using the auxiliary space, we solve a constraint energy minimization problem to construct offline multiscale spaces. The minimization problem is defined in the oversampling domain, which is larger than the target coarse block. To get a good approximation space, the oversampling domain should be large enough. We also propose a relaxed minimization problem to construct multiscale basis functions, which will yield more accurate and robust solution. To take into account the influence of input parameters, such as source terms, we propose the construction of online multiscale basis and an adaptive enrichment algorithm. We provide extensive numerical experiments on 2D and 3D models to show the performance of the proposed method.

AMS subject classifications: 65M99, 74B05, 65N30

Key words: Multiscale, elasticity, finite elements.

1 Introduction

In many science and engineer problems, one encounters multiple scales and high contrast. For example, wave propagation in fractured media, immiscible flow processes in poroelastic media and so on. Due to the advancement of media characterization methods and geostatistical modeling techniques, the media can be detailed at very fine scales,

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as a result, one needs to solve huge dimensional algebraic systems. Therefore, model reduction methods are proposed by researchers to reduce the problem size and alleviate the computational cost. Typical model reduction techniques include upscaling and multiscale methods. In upscaling methods [12, 17, 24], one typically upscales the media properties based on the homogenization theory so that the problem can be solved on a coarse grid. In multiscale methods [1, 4, 5, 11, 13, 14, 16, 18, 19, 23, 25], one still solves the problems on a coarse grid but with precomputed media dependent multiscale basis functions.

Among above mentioned multiscale methods, the multiscale finite element method (MsFEM) [14, 18] is a classic multiscale method that has shown great success in various practical applications. However, the MsFEM assumes that the media is scale separable. To overcome this assumption, the generalized multiscale finite element method (GMsFEM) [15] was proposed. The GMsFEM provide a systematic way to construct multiple multiscale basis. In particular in GMsFEM, one first creates an appropriate snapshot space and then solve a carefully designed local spectral problem in snapshot space. The basis space are filled with the dominant eigenvectors corresponding to small eigenvalues. The GMsFEM's convergence depends on decay behavior of the eigenvalues of the local spectral problems [15]. In [8], the authors applied the GMsFEM to solve the linear elasticity problem in high contrast problem, they consider both the continuous and discontinuous Galerkin method to couple the multiscale basis functions. In this paper, we will extend the recently proposed constraint energy minimizing GMsFEM (CEM-GMsFEM) [9] for high contrast linear elasticity problem. The CEM-GMsFEM consists of two steps. One needs to first construct auxiliary basis functions by solving local spectral problems on coarse element. Then, for each auxiliary basis function, one can construct a multiscale basis via energy minimization problems on subdomains that contains the support of the auxiliary basis. We propose two versions. The first one is based on solving constraint energy minimization problems and the second one is the relax version by solving unconstrained energy minimization problems. The CEM multiscale basis decays exponential outside the coarse element, which shares similar properties with the some localization methods such as the local orthogonal decomposition (LOD) and rough polyharmonic splines (RPS), see [20–22] The convergence of the CEM-GMsFEM not only depends on the eigenvalue but also depends on the coarse mesh size when the oversampling domain is carefully chosen.

To incorporate the influence of source and global media information, we also propose the construction of online multiscale basis. The idea of online approach was first proposed in [6] and has been extended to various other cases (see [3, 7, 26]). The key idea is using the residual information of the coarse-grid solution to construct multiscale basis. These online multiscale basis functions can also be computed adaptively so that the error can be decreased the most. The online basis of CEM-GMsFEM [10] will be computed in a oversampled domain, which is different from the original online approach [6]. We test our methods on 2D and 3D media with channels and inclusions. By properly selecting the number of basis functions and oversampling layers, we can observe that the multiscale