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A Multirate Approach for Fluid-Structure Interaction Computation with Decoupled Methods

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Abstract. We investigate a multirate time step approach applied to decoupled methods in fluid and structure interaction (FSI) computation, where two different time steps are employed for fluid and structure respectively. For illustration, the multirate technique is examined by applying the decoupled β scheme. Numerical experiments show that the proposed approach is stable and retains the same order of accuracy as the original single time step scheme, while with much less computational expense.

AMS subject classifications: 65M12, 37N10, 65N22, 65N30, 65M60, 65P99

Key words: Fluid and structure interaction (FSI), decoupled methods, multirate time step, stability, β scheme.

1 Introduction

Fluid structure interaction (FSI) finds many important applications in science and engineering [3–5, 7, 15–17, 25–27]. Numerical FSI methods may be generally classified as fully implicit and decoupled approaches. The fully implicit approach leads to coupled schemes [29], in which the equations of fluid dynamics, structural mechanics, and mesh moving are solved simultaneously in a fully coupled fashion. Although the coupled schemes are unconditionally stable in general, they usually result in significant difficulties and inflexibility in the design and choice of mesh generation, PDE discretization, algebraic solvers, as well as software development. On the other hand, certain decoupled approaches, often called loosely coupled, or partitioned, or explicit coupling approaches, have been investigated [1,2,13,20], where the equations of fluid dynamics, structural mechanics, and mesh moving are solved locally so that existing fluid and structure solvers may be applied by easy software integration. There are many other important physical

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and numerical considerations that appeal to decoupled methods for treating different sub-models in their own physical regions independently, particularly because the fluid and solid possess very different physical properties and time scales. This is generally true, not only for FSI problems [6, 11, 14–16, 27, 29], but also for other coupled multi-domain, multi-physics applications [8,9,18,19,21–24].

Due to different time scales in many multi-physics applications, it is natural and important to develop multirate time-stepping schemes that mimic the physical phenomena. A multirate time-step technique was introduced in [21,22] for decoupled methods of coupled fluid-porous media flow models, where the entire time interval $[0,T]$ is first partitioned into certain coarse time-grids with the time-step size τ_{coarse}. Within each coarse-time grid, the free fluid flow solutions are computed for multiple fine-time steps with the boundary information at the interface from the porous medium solution at the beginning of the current coarse-time grid. When the computation reaches the end of current coarse-time grid, the porous medium solution is then updated by using the data from the fluid region. Such a multirate approach is proved to be stable. Other multi-domain, multi-physics applications which adopt multirate time-step technique can be found in [23,24].

We propose to develop multirate decoupled algorithms for FSI applications in this paper. It has been observed that decoupled methods might lead to instability for certain FSI applications, such as the coupling of an incompressible Stokes flow with a thin-walled structure, due to the artificial added-mass effect if the coupled FSI model is not properly decoupled [10, 12]. It has become one of the major challenges recently to develop stable decoupled methods without added-mass effects for such applications. For the benchmark problem of the coupled incompressible Stokes flow with a thin-walled structure, two decoupled methods have been devised recently, one is the Robin-Neumann scheme [11] and the other is the so-called β scheme [6]. Both algorithms are shown to be stable without added-mass effects [6, 11].

Note that the structure variables vary much more rapidly than the fluid variables in this application. We thus propose to apply the multirate time-stepping to these stable decoupled schemes. For illustration, we will examine the application of the multirate technique to the β scheme, since similar performance is observed for both the Robin-Neumann scheme and the β scheme in numerical experiments. Numerical experiments demonstrate that the proposed approach is stable and retains the same order of accuracy as the original single time step schemes, while with much less computational expense. Furthermore, the decoupled multirate β scheme may be extended to more general FSI problems involving nonlinearity, irregular domains, and large structural deformations, which is illustrated by a computational biomechanical model for abdominal aneurysm simulation.

The paper is organized as follows. In Section 2, we describe the coupled model of a Stokes flow with a thin-walled structure. In Section 3, the multirate β scheme is presented for the coupled model. Numerical experiments are presented in Section 4 to show the stability, convergence, and efficiency of the multirate β scheme. Concluding remarks are given in Section 5.