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An Adaptive Moving Mesh Discontinuous Galerkin Method for the Radiative Transfer Equation

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Abstract. The radiative transfer equation models the interaction of radiation with scattering and absorbing media and has important applications in various fields in science and engineering. It is an integro-differential equation involving time, frequency, space and angular variables and contains an integral term in angular directions while being hyperbolic in space. The challenges for its numerical solution include the needs to handle with its high dimensionality, the presence of the integral term, and the development of discontinuities and sharp layers in its solution along spatial directions. Its numerical solution is studied in this paper using an adaptive moving mesh discontinuous Galerkin method for spatial discretization together with the discrete ordinate method for angular discretization. The former employs a dynamic mesh adaptation strategy based on moving mesh partial differential equations to improve computational accuracy and efficiency. Its mesh adaptation ability, accuracy, and efficiency are demonstrated in a selection of one- and two-dimensional numerical examples.

AMS subject classifications: 65M50, 65M60, 65M70, 65R05, 65.75

Key words: Adaptive moving mesh, discontinuous Galerkin method, radiative transfer equation, high order accuracy, high resolution.

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1 Introduction

The radiative transfer equation (RTE) models the interaction of radiation with scattering and absorbing media, which has important applications in fields such as astrophysics, high energy density physics, nuclear physics, inertial confinement fusion, heat transfer, stellar atmospheres, optical molecular imaging, infrared and visible light in space and the atmosphere, and biomedicine. The RTE is an integro-differential equation with seven independent variables (i.e., time, frequency, space, and angles) for a time-dependent three-spatial-dimensional problem. Containing an integral term and with its high dimensionality, the RTE presents a challenge in the development of efficient numerical algorithms. On the other hand, the efficient solution of the RTE plays an important role in the study of radiation hydrodynamics where the RTE is often coupled with the Euler equations, the energy equation, and the equation of state.

In the past, a number of methods have been developed for the numerical solution of the RTE. Those methods can be divided roughly into two categories: stochastic and deterministic approaches. The Monte Carlo method is a widely used method in the former category [24, 31]. On the other hand, deterministic approaches involve discrete approximations of the variables in the RTE. In particular, the discretization needs to be applied to all coordinates in space and angles. For angular coordinates, the P_N method, first introduced in [25] and also known as the spherical harmonics method, uses an orthogonal, harmonic basis to approximate the solution. Another approach called the discrete ordinate method (DOM) [6, 26] employs spectral collocation and the Legendre-Chebyshev quadrature to discretize the integro-differential equation in angular coordinates. DOM is widely used for the numerical solution of the transport equation [28, 35] due to its high accuracy, flexibility, and relatively low computational cost. The angle-discretized RTE forms a system of linear hyperbolic equations with a numerical integral term, which can be discretized in space using a standard method such as a finite difference, finite volume, or finite element method. The discontinuous Galerkin (DG) method is employed for this purpose in the current work.

The DG method is known to be a particularly powerful numerical tool for the simulation of hyperbolic transport problems. It was first used for the RTE by Reed and Hill [36] and theoretically studied by Lesaint and Raviart [27]. The method was later extended to nonlinear conservation laws by Cockburn and Shu [7–10]. The DG method has the advantages of high-order accuracy, geometric flexibility, suitability for handling h- and p-adaptivity, extremely local data structure, high parallel efficiency, and a good theoretical foundation for stability and error estimates. Over the last few decades, the DG method has been used widely in scientific and engineering computation.

The objective of the current work is to study an adaptive moving mesh DG method (for spatial discretization) combined with DOM (for angular discretization) for the numerical solution of the RTE. Due to its hyperbolic nature, the solution of the RTE can develop discontinuities or sharp layers along spatial directions, which makes mesh adaptation an indispensable tool for use in improving computational accuracy and efficiency.