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An Interface-Unfitted Conforming Enriched Finite Element Method for Stokes-Elliptic Interface Problems with Jump Coefficients

Hua Wang[1], Jinru Chen[2], Pengtao Sun[3,*] and Rihui Lan[3]

[1] *School of Mathematical Sciences, Peking University, Beijing 100871, China.*
[2] *Jiangsu Key Laboratory for NSLSCS, & School of Mathematical Sciences, Nanjing Normal University, Jiangsu 210023, China.*
[3] *Department of Mathematical Sciences, University of Nevada Las Vegas, 4505 Maryland Parkway, Las Vegas, NV 89154, USA.*

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Abstract. In this paper, a conforming enriched finite element method over an interface-unfitted mesh is developed and analyzed for a type of Stokes-elliptic interface problem with jump coefficients. An inf-sup stability result that is uniform with respect to the mesh size is proved in order to derive the corresponding well-posedness and optimal convergence properties in spite of the low regularity of the problem. The developed new finite element method breaks the limit of the classical immersed finite element method (IFEM) which can only deal with the case of identical governing equations on either side of the interface. Numerical experiments are carried out to validate the theoretical results. This is the first step of our new method to solve complex interface problems with different governing equations on either side of the interface, and will be extended to solve transient interface problems towards fluid-structure interaction problems in the future.

AMS subject classifications: 65M60

Key words: Conforming enriched finite element, interface-unfitted mesh, Stokes-elliptic interface problem, inf-sup condition, optimal convergence.

1 Introduction

In this paper we consider the following Stokes-elliptic interface problem in a bounded connected convex polygonal domain Ω in $\mathbb{R}^2$, and Ω is separated by a C^2-continuous

*Corresponding author. *Email addresses:* `wanghua.math@foxmail.com` (H. Wang), `jrchen@njnu.edu.cn` (J. Chen), `pengtao.sun@unlv.edu` (P. Sun), `lanr1@unlv.nevada.edu` (R. Lan)

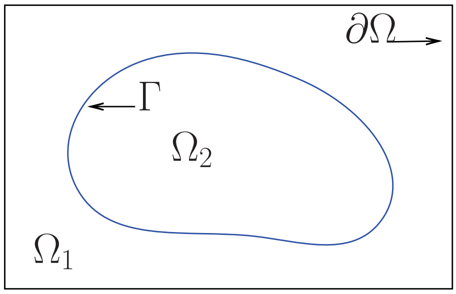

Figure 1: A sketch of the domain for the Stokes-elliptic interface problem.

interface Γ into two sub-domains Ω_1 and Ω_2 (see Fig. 1 for an illustration),

$$-\nabla\cdot(\beta_1\nabla \boldsymbol{u}_1)+\nabla p_1=\boldsymbol{f}_1 \quad \text{in } \Omega_1, \tag{1.1}$$
$$\nabla\cdot\boldsymbol{u}_1=0 \quad \text{in } \Omega_1, \tag{1.2}$$
$$-\nabla\cdot(\beta_2\nabla \boldsymbol{u}_2)=\boldsymbol{f}_2 \quad \text{in } \Omega_2, \tag{1.3}$$
$$\boldsymbol{u}_1=\boldsymbol{u}_2 \quad \text{on } \Gamma, \tag{1.4}$$
$$(\beta_1\nabla\boldsymbol{u}_1-p_1\boldsymbol{I})\boldsymbol{n}_1+\beta_2\nabla\boldsymbol{u}_2\boldsymbol{n}_2=\boldsymbol{g} \quad \text{on } \Gamma, \tag{1.5}$$
$$\boldsymbol{u}_1=\boldsymbol{0} \quad \text{on } \partial\Omega_1\backslash\Gamma, \tag{1.6}$$

where, the jump coefficients $\beta_1\neq\beta_2$, $\boldsymbol{n}_i$ denotes the unit outward normal vector of Ω_i on Γ, $i=1,2$.

The Stokes-elliptic interface problem (1.1)-(1.6) can be roughly viewed as a static linearized fluid-structure interaction problem. Fluid-structure interactions (FSI) [10,14,24–27] is the interaction of some movable or deformable structures with an internal or surrounding fluid flow, which play prominent roles in many scientific and engineering fields, yet a comprehensive study of such problems remains a challenge due to their strong nonlinearity and multidisciplinary nature. For most FSI problems, analytical solutions to the model equations are impossible to be obtained, whereas laboratory experiments are limited in scope. Thus to investigate the fundamental mathematics and physics involved in the complex interaction between fluid equations and structure equation, numerical method must be developed and necessarily analyzed to guarantee its reliability and accuracy. In this paper, we start with a simplified static model first as shown in (1.1)-(1.6), where, β_1 may stand for the fluid viscosity, and β_2 for the elastic parameter of the structure. In practice, different constitutive relations corresponding to the fluid and the structure shall be employed to reformulate their equations in (1.1) and (1.3) for FSI problems, respectively. The interface condition of jump flux (1.5) needs to be redefined as well. Hence, the essential characteristic of a static FSI model is preserved in the static Stokes/elliptic interface problem (1.1)-(1.6), that is, two different types of governing equations bearing with different primary unknowns and different compressibility/constitutive relation are defined on either side of the interface Γ.

We use boldface characters to denote vector-valued or matrix-valued functions. For