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Evaluation of Selected Finite-Difference and Finite-Volume Approaches to Rotational Shallow-Water Flow

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Abstract. The shallow-water equations in a rotating frame of reference are important for capturing geophysical flows in the ocean. In this paper, we examine and compare two traditional finite-difference schemes and two modern finite-volume schemes for simulating these equations. We evaluate how well they capture the relevant physics for problems such as storm surge and drift trajectory modelling, and the schemes are put through a set of six test cases. The results are presented in a systematic manner through several tables, and we compare the qualitative and quantitative performance from a cost-benefit perspective. Of the four schemes, one of the traditional finite-difference schemes performs best in cases dominated by geostrophic balance, and one of the modern finite-volume schemes is superior for capturing gravity-driven motion. The traditional finite-difference schemes are significantly faster computationally than the modern finite-volume schemes.

AMS subject classifications: 76U05, 86A05, 35L65, 65M08

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1 Introduction

In this paper, we examine four different numerical schemes for the shallow-water equations in a rotating frame. These equations are important for a range of application areas, including simulation of the ocean and atmosphere. We focus on oceanographic simulations, in which the equations can capture the short-term ocean dynamics that are important for e.g., storm surge predictions. Our aim is to evaluate the suitability of the numerical schemes for use in an ensemble prediction system with data assimilation. One example is the propagation of long waves in an ocean basin sufficiently large so that the motion is constrained by geostrophy, and where we need to consider the effects of topography and nonlinearity, e.g. in the Barents Sea, which is fairly shallow but have large tidal range. We emphasize that we do not seek realistic solutions of the ocean dynamics for specific regions here, but rather aim to compare the various schemes using a range of parameters relevant for such dynamics.

In the early days of computational oceanography, finite-difference schemes were popular to simulate the rotational shallow-water equations. With increasing computational power, more complex physics, grids, and new discretization methods have appeared. Today's state-of-the-art ocean circulation models are sophisticated 3D simulations that capture a lot of the physical driving forces of the ocean currents, yet these models are computationally demanding and therefore allow only a limited number of ensemble members to be run in reasonable time.

We revisit two finite-difference schemes from early computational oceanography and compare these against two modern finite-volume schemes. One of our motivations for comparing finite-volume and finite-difference discretizations is that there has been recent developments in the finite-volume community for the rotating shallow-water equations, and we want to evaluate these from a cost-benefit perspective against well-known models.

The dominant force balance in the equations implies a nonzero current as a steady state, as the pressure gradient needs to be balanced by the Coriolis forces (so-called geostrophic balance). This is very different from problems in which the Earth's rotation can be ignored, where a typical steady state would imply zero velocities, often referred to as "lake-at-rest". This difference has important implications for the discretization of numerical schemes, and has been one of the driving factors in the development of modern high-resolution finite-volume methods which are well-balanced according to such steady-state solutions.

The four selected schemes in this work are all based on Cartesian grids, and are selected both because they capture the important geostrophic balance required for short-term predictions, and because they are very well suited for implementation on the GPU. Our long-term goal is to run large ensembles of such models, initialized and downscaled from operational 3D circulation models, on the GPU. These large ensembles can then be used on-demand to provide uncertainty estimates in predictions of storm surge or in drift trajectory modelling. The four numerical schemes we examine are: