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# Adaptive Anisotropic Unstructured Mesh Generation Method Based on Fluid Relaxation Analogy

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**Abstract.** In this paper, we extend the method (Fu et al., [1]) to anisotropic meshes by introducing an adaptive SPH (ASPH) concept with ellipsoidal kernels. First, anisotropic target feature-size and density functions, taking into account the effects of singularities, are defined based on the level-set methodology. Second, ASPH is developed such that the particle distribution relaxes towards the target functions. In order to prevent SPH particles from escaping the mesh generation regions, a ghost surface particle method is proposed in combination with a tailored interaction strategy. Necessary adaptations of supporting numerical algorithms, such as fast neighbor search, for enforcing mesh anisotropy are addressed. Finally, unstructured meshes are generated by an anisotropic Delaunay triangulation conforming to the Riemannian metrics for the resulting particle configuration. The performance of the proposed method is demonstrated by a set of benchmark cases.

**AMS subject classifications**: 65M50, 51F99, 68U05, 34C40, 65M60, 65E05

**Key words**: Adaptive unstructured meshes, anisotropic meshes, level-set, SPH, anisotropic Delaunay triangulation.

## 1 Introduction

Anisotropic unstructured meshes conforming to given Riemannian metric tensor fields, which often are constructed from solution error estimates [2–4], are widely used in scientific computing [5–7]. In many engineering applications, fields that are to be discretized by meshes vary in time and space, and may exhibit strong anisotropy. E.g. boundary layer flows have distinct evolution in streamwise and wall-normal directions [8, 9]. For such scenarios, adaptive anisotropic meshes, consisting of triangles elongated along preferred orientations, are much preferable to isotropic meshes [10, 11]. High-quality

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anisotropic unstructured mesh generation also is essential for surface modeling [12, 13], image processing [14], and function interpolation [15]. Compared to isotropic meshes, anisotropic meshes are more difficult to generate since size, shape and orientation of mesh elements should be taken into account simultaneously. The anisotropic mesh quality highly depends on the distribution of vertices. Algorithms proposed for isotropic mesh generation have been generalized to adaptive anisotropic mesh generation [16, 17], some of them available as open-source libraries, e.g. MMG3D [18] and Feflo.a [19].

A point placement algorithm based on the advancing-front concept has been developed by Marcum and Alauzet [20, 21] to align mesh elements with a solution based metric field and has been successfully applied to complex interface-tracking problems, e.g. shock-bubble interactions. The Delaunay-type mesh generation algorithm with incremental point-insertion process has been validated to work effectively for anisotropic mesh generation through replacing the usual metric by a Riemannian metric [22, 23]. Based on the local point insertion, e.g. Delaunay insertion of Steiner points [24], Dobrzynski and Frey [25] propose a convergent algorithm to adapt mesh elements to an anisotropic metric tensor locally. It is demonstrated that irregularly-shaped elements can be prevented by slightly modifying the Delaunay kernel. However, the Delaunay criterion is fulfilled only locally rather than globally.

With the definition of a directional distance function as simplification of the classical Riemannian distance measure, the Centroidal Voronoi tessellations (CVT) mesh generation method in the Euclidean space is extended to Anisotropic CVT (ACVT) with respect to a given Riemannian metric [26]. The corresponding definitions of Anisotropic Voronoi region (AVR), Anisotropic Voronoi Tessellation (AVT) and Anisotropic Delaunay triangulation (ADT) lead to a consistent description of mass centroids as well as ACVT. ACVT reverts to standard CVT when the Riemannian metric degenerates to an isotropic tensor. However, since the anisotropic Delaunay triangulation conforming to the Riemannian metrics must be reconstructed for each Lloyd-iteration step, the ACVT method is computationally expensive [26, 27]. Some discrete approximate algorithms for the ACVT construction have been proposed at the expense of quality degeneration [28]. Moreover, *orphan subregions*, which may corrupt the result of ACVT, cannot be fully avoided if not a certain minimum number of generators is involved [26].

Bossen and Heckbert [29] develop a flexible mesh generation algorithm, in which mesh vertices are repositioned according to attraction/repulsion with their neighbors and for which the Delaunay triangulation is maintained for convenient neighbor search, vertex insertion and deletion. Anisotropic Delaunay triangulation at each iteration renders it time-consuming and complex. Furthermore, an inadequate guess of initial particle quantity to fill the domain may induce convergence problems, and a mesh quality conforming to the anisotropic metrics is not guaranteed. Yamakawa and Shimada [30] propose the "bubble packing" anisotropic tetrahedral meshing method. Ellipsoidal bubbles are first packed in the domain and then iteratively migrated to a stable state according to a mass-spring-damper model. At last, the mesh is reconstructed using the advancing-front method. By first mapping the anisotropic space into a higher-dimensional isotropic