

A Study of Several Artificial Viscosity Models within the Discontinuous Galerkin Framework

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Abstract. Dealing with strong shocks while retaining low numerical dissipation traditionally has been one of the major challenges for high order methods like discontinuous Galerkin (DG). In the literature, shock capturing models have been designed for DG based on various approaches, such as slope limiting, (H)WENO reconstruction, a posteriori sub-cell limiting, and artificial viscosity, among which a subclass of artificial viscosity methods are compared in the present work. Four models are evaluated, including a dilation-based model, a highest modal decay model, an averaged modal decay model, and an entropy viscosity model. Performance for smooth, non-smooth and broadband problems are examined with typical one- and two-dimensional cases.

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1 Introduction

Computational fluid dynamics (CFD) is increasingly involved in the design process of modern aircrafts. High order methods are essential for various problems, such as turbulence flows, acoustic prediction, and long time convection of vortices from a wingtip [1], all of which pose a stringent limit on the error level of the simulation, and for which high order methods have been proved to be more efficient than their second order counterpart [2]. The last decades have seen an explosive growth in the amount of research activity in high order methods, among which discontinuous Galerkin (DG) methods [3] have been considered to have great potential in terms of accuracy, geometrical flexibility, h-p adaptivity, parallel efficiency, and so on.

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One of the major challenges for high order methods is to handle discontinuous solutions with a reasonable balance between accuracy and robustness. For methods like DG, various types of shock capturing methods have been proposed in the literature. The first approach is slope limiting, such as in [4, 5]. Despite its total variation bounded (TVB) property, this method has a detrimental effect on accuracy especially for higher order accuracy, and depends strongly on empirical parameters. Improvements on these issues have been proposed [6–8]. However, it remains difficult to retain high order accuracy with limiting. An alternative strategy is a limiter based on the weighted essentially non-oscillatory (WENO) reconstruction, which first identifies cells close to discontinuities as troubled cells, after which a WENO reconstruction is performed. The first such ideas were proposed in [9, 10], which use only the cell averages for the reconstruction and therefore requires quite large stencils. To relieve this issue, Hermite WENO (HWENO), using both the cell average and its first derivatives, was proposed in [11] and extended to two-dimensional problems in [12, 13]. [14] improves the HWENO method to use also the derivatives of higher than first order to reduce the stencil to the immediate neighborhood. However, only results for the second order case have been demonstrated. To reduce the stencil size, the key is to make use of as much information as possible in each cell. Following this idea, [15, 16] proposes a simple yet effective WENO limiting method which reconstructs the entire polynomial instead of just point-wise values or moments. This results in a stencil consisting of only immediate neighboring cells, and eliminates the issue of negative weights. Furthermore, the method is able to achieve uniform high order accuracy, although no results for higher than $P3$ have been demonstrated. In [17, 18], the authors proposed a subcell limiter based on the MOOD algorithm [19, 20], which a posteriori verifies the validity of a discrete candidate solution against some physical and/or numerical criteria after each time step. The MOOD-based subcell limiter can be extended to high orders, and is equipped with the maximum principle preservation or positivity preservation. Following this line, an improved version of a posteriori correction was developed in [21]. More limiters for DG can be found in [22–26].

This paper focuses on another family of methods, capturing discontinuities by adding explicit dissipation to the original system. The key for these methods is to develop sensors which measure the smoothness of the flow field and determine the required amount of artificial dissipation. One idea is to employ derived quantities as the shock sensor, as was initiated in the context of finite difference methods in [27, 28], and later improved in [29–31]. The method tackles vortices, shocks, and contact discontinuities with artificial shear viscosity, artificial bulk viscosity, and artificial thermal conductivity, respectively. Further, [32] extends the method to unstructured grids with the spectral difference (SD) method, and [33] simplifies the method based on a scaling function of velocity dilation using the hybridizable discontinuous Galerkin (HDG) scheme. Such methods are quite robust and can handle complicated flows with great flexibility since they design specific sensors for different flow features. However, they introduce a general dilemma of choosing the order of the derived quantities. For higher derivatives, accuracy in smooth regions is better, while the complexity and cost for computing the derivatives are high