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Improved RBF Collocation Methods for Fourth Order Boundary Value Problems

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Abstract. Radial basis function (RBF) collocation methods (RBFCMs) are applied to fourth order boundary value problems (BVPs). In particular, we consider the classical Kansa method and the method of approximate particular solutions (MAPS). In the proposed approach we include some so-called ghost points which are located inside and outside the domain of the problem. The inclusion of these points is shown to improve the accuracy and the stability of the collocation methods. An appropriate value of the shape parameter in the RBFs used is obtained using either the leave-one-out cross validation (LOOCV) algorithm or Franke's formula. We present and analyze the results of several numerical tests.

AMS subject classifications: 65N35, 65N99

Key words: Radial basis functions, Kansa method, method of particular solutions, collocation, fourth order PDEs.

1 Introduction

In recent years, meshless methods have undergone vigorous development and have matured as methods of choice for the solution of various science and engineering problems. Unlike other traditional mesh-based methods such as the finite element method [1,2,25], the finite difference method [34,35], and the finite volume method [22,23], the main attraction of meshless methods is their ability to easily and effectively solve problems in

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complex geometries, particularly in high dimensions. There are various types of meshless methods and among them, radial basis function (RBF) collocation methods (RBFCMs) have become increasingly popular and attracted considerable attention in recent years for solving partial differential equations (PDEs). The first RBFCM was proposed by Kansa in 1990 [17] and, over the years, various kinds of RBFCMs, such as the method of approximate particular solutions (MAPS) [3], the RBF-differential quadrature method [32], collocation for Cauchy problems [24], and the RBF Hermite collocation method [8], have been proposed. Note that each of the above RBFCMs has its own advantages and disadvantages. Undoubtedly, the most famous and popular RBFCM is the Kansa method which we shall be investigating in the current study. The Kansa method is known to produce highly accurate results but is sensitive to the RBF shape parameter values and its stability can be an issue. Among all of the above RBFCMs, the MAPS is an alternative indirect method closely related to the Kansa method and has also proved to be effective. Unlike the Kansa method, the MAPS adopts a particular solution of the considered differential operator, with respect to the RBF, as the "new" basis function. The derivation of a particular solution comes from an integration process and the obtained approximation is usually more stable with respect to the shape parameter. In the last few years, the MAPS has attracted some attention in the RBF community and has been applied to solve a large class of PDEs. Despite the success of the MAPS, the derivation of a particular solution in closed-form for general differential operators remains a major challenge. For high order differential operators, the derivation of a closed-form particular solution is substantially more difficult to obtain. Once the closed-form particular solution for a given basis function and differential operator is available, the MAPS converts the given PDE into an interpolation problem. Due to the difficulty of deriving a particular solution for a general differential operator, an alternative is the use of the particular solution of the Laplace operator for second order PDEs and the biharmonic operator for fourth order PDEs, as the basis function. This approach for solving fourth order PDEs will be adopted in this paper. It should be mentioned that often, in order to save computational effort in problems requiring a large number of nodes, global RBFs may be replaced by local RBF methods [4,6,21,38].

A major challenge in RBFCMs is the determination of an optimal, or at least suitable, value of the shape parameter in the RBF used. Various studies have attempted to overcome this difficulty, see, for example, [16,18,19,29]. We shall use the leave-one-out cross validation algorithm (LOOCV) [10,29] and a modification of Franke's formula [12]. The issue of determining an appropriate value of the shape parameter can be avoided by using polyharmonic splines with additional polynomials which gives comparable accuracy to infinitely differentiable RBFs, see, e.g. [11,16,28,37].

Fourth order boundary value problems (BVPs) are important in modelling various types of physical problems such as plate bending [13], fluid dynamics [15], and computer graphics problems [33], to name a few. Many meshless methods have been developed for the solution of fourth order PDEs where one of the challenges is that two boundary conditions need to be imposed. As such, the resulting matrix in the classical