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A Two-Dimensional Second Order Conservative Front-Tracking Method with an Original Marker Advection Approach Based on Jump Relations

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Abstract. A two-dimensional front-tracking method is developed for handling complex shape interfaces satisfying the volume conservation. In order to validate the proposed front-tracking method, a complete convergence study is carried out on several analytical test cases for which the interface is widely stretched and deformed. Comparisons to different existing approaches show that our front-tracking method is second order accurate in space with lower errors than existing interface tracking techniques of the literature.

We also propose an original marker advection method which takes into account the jump relations valid at interface in order to deal with the contrast of physical properties encountered in two-phase flow simulations. The conservative front-tracking method computed in this work is shown to be able to describe interfaces with high accuracy even for poorly resolved Eulerian grids.

AMS subject classifications: 76

Key words: Front-tracking, marker velocity reconstruction based on jump relations, multiphase flow, volume conservation, second order accuracy.

1 Introduction

In past two decades, major progress has been achieved in fluid interface tracking when multiphase flow simulations are tackled with. Various tracking techniques have been investigated in the literature to handle these flow simulations. Depending on how the

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pressure and velocity variables are computed and how the interface is taken into account in the conservation equations, either explicitly with a moving interfacial mesh or indirectly by capturing the presence of the interface through an auxiliary variable, these tracking techniques can be divided in two major classes:

- On one hand, moving unstructured body fitted meshes, pure Lagrangian [17] or arbitrary Eulerian-Lagrangian methods [13, 14] are techniques that conform to the interfacial shape. They use the two-fluid flow equations for simulating multiphase flows, *i.e.* the Navier-Stokes equations in each phase and jump relations on the mesh elements of the interface [8]. Indeed, the deformations of the interface are explicitly tracked over time with deformable meshes which follow the front surface. Quite popular for fluid structure interaction problems, ALE approaches are widely used in this class of tracking methods for free surface flow problems since they avoid the fast mesh distortion known from pure Lagrangian methods. Thanks to the explicit representation of the interface, surface characteristics such as the surface tension of the material properties can be computed accurately. However, the management of the moving deformable mesh involves high complexity and limits the application of this approach to two dimensional configurations or weakly deformable interfaces in three dimensions. In the range of body fitted approaches with mesh points describing the interface, the adaptive curvilinear orthogonal grid techniques developed in Jadim code are also interesting to mention [28]. For example, they allow the accurate simulation of single droplet or bubble flows in axisymmetric coordinates.

- On the other hand, another class of approach to track interfaces, generally on fixed Eulerian meshes, consists in capturing directly the presence of the front by using an auxiliary Eulerian variable. The pioneer work of Harlow and Welch [18] deals with the tracking of Lagrangian volume parcels of fluid in order to describe the evolution of one of the two existing phases on a structured grid. In these approaches, the interface is not *a priori* known, only its presence in a given cell is reported. Among the huge amount of works devoted to these interface capturing techniques, we can cite the Volume Of Fluid (VOF) method [59], for which the interface is represented by means of a phase function. This is one of the oldest but still popular front-capturing approach as it provides naturally volume conservation of phases. Another approach that is very often used by the scientific community is the level-set method [50]. With this method, the interface is located by a signed distance function which is continuous across the interface and allows an accurate estimate of the curvature and normal at the interface. A combination of both level set and VOF methods has been proposed by various authors [5, 10, 49] to ensure a mass conservative tracking approach while preserving the simple definition and implementation of the level-set method. Even if the research has been focused on fixed regular grids, more recent developments [6, 9, 22, 32, 58] allowed the extension of these methods to three dimensional unstructured meshes for VOF or level set meth-