Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2018-0087

Vol. **28**, No. 1, pp. 228-248
July 2020

Subspace Methods in Multi-Parameter Seismic Full Waveform Inversion

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Received 2 April 2018; Accepted 6 March 2019

Abstract. In full waveform inversion (FWI) high-resolution subsurface model parameters are sought. FWI is normally treated as a nonlinear least-squares inverse problem, in which the minimum of the corresponding misfit function is found by updating the model parameters. When multiple elastic or acoustic properties are solved for, simple gradient methods tend to confuse parameter classes. This is referred to as parameter cross-talk; it leads to incorrect model solutions, poor convergence and strong dependence on the scaling of the different parameter types. Determining step lengths in a subspace domain, rather than directly in terms of gradients of different parameters, is a potentially valuable approach to address this problem. The particular subspace used can be defined over a span of different sets of data or different parameter classes, provided it involves a small number of vectors compared to those contained in the whole model space. In a subspace method, the basis vectors are defined first, and a local minimum is found in the space spanned by these. We examine the application of the subspace method within acoustic FWI in determining simultaneously updates for velocity and density. We first discuss the choice of basis vectors to construct the spanned space, from linear updates by distinguishing only the contributions of different parameter classes towards nonlinear updates by adding the contributions of higher-order perturbations of each parameter class. The numerical character of FWI solutions generated via subspace methods involving different basis vectors is then analyzed and compared with traditional FWI methods. The subspace methods can provide better reconstructions of the model, especially for the velocity, as well as improved convergence rates, while the computational costs are still comparable with the traditional FWI methods.

AMS subject classifications: 35L05, 35R30, 86A22

Key words: Waveform inversion, inverse problem, subspace method.

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1 Introduction

In full waveform inversion (FWI) [11, 25, 27], subsurface model parameters are determined by minimizing an objective function measuring the difference between predicted data and recorded data, related through forward modelling. The forward modelling involves wave propagation physics which can range from scalar acoustic, to acoustic, up to viscoelastic anisotropic approximations, and beyond. Simultaneous inversion for different parameter classes (see, e.g., [3, 6, 16, 17, 19–21]), including for instance P-wave and S-wave velocities, density, as well as the various attenuation and anisotropic parameters, etc., are critical for a wide application of FWI in reservoir characterization. Similar to the mono-parameter inversion under the scalar acoustic approximation, in which only P-wave velocity is considered, in multi-parameter FWI a misfit function is set up to describe the distance between the recorded data and the predicted data, and FWI is treated as a nonlinear least squares problem, which can be solved by gradient-based methods or Newton-type methods. Multi-parameter inversion is more complicated than mono-parameter inversion, because the additional parameter classes increase the ill-posedness and the nonlinearity of the inverse problem. Different parameter classes can be more or less coupled, and it may be difficult to distinguish the contribution of each parameter class to changes in the data. Mitigating cross-talk between different parameter classes is a key issue. Studies have shown that the Hessian operator contains some information concerning the coupling between different parameter classes. Different ways of incorporating the inverse of the Hessian operator, especially in multi-parameter inversion, have been proposed to better decouple different parameter classes in the inversion. These include preconditioning the gradient using the pseudo Hessian matrix [24], quasi-Newton method, truncated Newton method [12–15, 18] and so on. Hierarchical strategies can be applied to successively invert different parameter classes to mitigate the ill-posedness of FWI [1, 2, 9]. In most cases involving incorporation of Hessian information, significant increases in computational cost ensue.

In both gradient-based methods and Newton-type methods (see, e.g., [7, 12, 13, 18, 22, 27, 31]), a line search scaling the descent direction tends to be necessary for convergence. One scalar is found for all parameter classes regardless of their contributions to the data. To combat cross-talk, distinguishing between the contributions of each parameter class during updating could be helpful in multi-parameter inversion.

Application of subspace methods in large-scale inverse problems was first discussed by [10,23] as an approach to adjusting the update descent directions according to different parameter classes' contributions. Baumstein [5] showed that using an extended subspace method in multi-parameter inversion can also help to mitigate cross-talk. In subspace FWI, basis vectors are determined first, and the optimization problem is then solved in this spanned space to minimize the quadratic approximation of the misfit function, with only a few coefficients to be determined as compared to the traditional gradient-based or Newton-type methods. Although projection of the full Hessian or Gauss-Newton Hessian onto the subspace is needed for each iteration, the calculation is much cheaper com-