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Concepts in the Direct Waveform Inversion (DWI) Using Explicit Time-Space Causality

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Abstract. The Direct Waveform Inversion (DWI) is a recently proposed waveform inversion idea that has the potential to simultaneously address several existing challenges in many full waveform inversion (FWI) schemes. A key ingredient in DWI is the explicit use of the time-space causality property of the wavefield in the inversion which allows us to convert the global nonlinear optimization problem in FWI, without information loss, into local linear inversions that can be readily solved. DWI is a recursive scheme which sequentially inverts for the subsurface model in a shallow-to-deep fashion. Therefore, there is no need for a global initial velocity model to implement DWI. DWI is unconditionally convergent when the reflection traveltime from the boundary of inverted model is beyond the finite recording time in seismic data. In order for DWI to work, DWI must use the full seismic wavefield including interbed and free surface multiples and it combines seismic migration and velocity model inversion into one process. We illustrate the concepts in DWI using 1D and 2D models.

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1 Introduction

The seismic full waveform inversion (FWI) [1, 2] initially formulated in the time domain represents an important conceptual leap whose purpose is to find a subsurface model that can be used to predict observed seismic waveforms in both phase and amplitude, wiggle to wiggle. FWI can also be implemented in the frequency domain [3–6]. Despite some success, FWI mathematical formulation has significant physical limitations. The goal of this paper is to understand the cause of the limitations and propose a new formulation, called direct waveform inversion (DWI), to overcome these limitations. At present, DWI

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is by no means perfect but nevertheless it provides promising directions for waveform inversion.

Seismic FWI problem was frequently cast into a global nonlinear mathematical optimization problem where a model is sought to minimize a misfit/objective function defined between the observed data and the model-predicted data [7]. The nonlinearity arises because the change in seismic data in response to the change in the model parameter is not linear. In FWI, one linearizes the problem around a starting model and then computes the local gradient (e.g., the Frechet derivative) of the data perturbation with respect to the model perturbation and updates the model along the gradient direction. The updated model will be the next starting model and this process can be iterated until certain criterion about the misfit is met.

Challenges to implement FWI were almost immediately recognized since the inception of the FWI idea [1,2]. The first one is the initial-model dependence and convergence issue in the nonlinear global optimization. The second one is FWI's apparent lack of ability to recover low-wavenumber (large-scale) strong-contrast model variations.

FWI results strongly depend on the initial model that is usually not an outcome of FWI itself but is provided as an input to FWI. FWI works well if a good initial model in the neighborhood of the true model can be found in the beginning [8–11]. If the initial model is far from the true model, the FWI iteration may converge to a local minimum of the objective function and the global optimization cannot be attainable. Real geological models are likely to be complex. Demanding an initial model that is already close to the true model undercuts the true value of practical implementation of FWI. Fortunately, Kolb et al. [12] showed in numerical examples that if a coarsely smoothed version of the true model is available as the starting model, FWI could converge to the true model. This conclusion had been confirmed more recently [4, 13]. *Can FWI produce its own low-wavenumber initial modeling?* Within the gradient-based FWI theoretical framework, we need low frequency seismic data in order for FWI to recover the low-wavenumber model component [12,14] because the FWI formulation/approximation is more linear/accurate at low frequencies. Seismic data in exploration settings are bandlimited. However, even without the low frequency data, the low-wavenumber model information is indeed contained in data and we can readily obtain it using many other methods such as traveltime tomography or the normal moveout analysis [15]. The inability of FWI to invert for low-wavenumber model variations relative to a simple starting model (e.g., homogeneous, or linear) shows the deficiency in FWI formulation.

Gradient-based FWI methods do not account for the full physics of wave scattering and propagation. Tarantola [16, p.128] pointed out that the FWI local Frechet gradient amounts to the linear single-scattering Born approximation. Recent work by Wu and Zheng [17] showed a one-to-one correspondence between the n-th order Frechet derivative and the n-th order multiple Born scattering. This means that FWI's dropping high-order functional derivatives is to physically ignore possible multiple scattering among unknown model perturbations/scatterers. This is a significant drawback in the FWI assumption. Wu and Zheng [17] further showed in numerical modeling that including