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Towards a Theoretical Background for Strong-Scattering Inversion – Direct Envelope Inversion and Gel′fand-Levitan-Marchenko Theory

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Abstract. Strong-scattering inversion or the inverse problem for strong scattering has different physical-mathematical foundations from the weak-scattering case. Seismic inversion based on wave equation for strong scattering cannot be directly solved by Newton's local optimization method which is based on weak-nonlinear assumption. Here I try to illustrate the connection between the Schrödinger inverse scattering (inverse problem for Schrödinger equation) by GLM (Gel′fand-Levitan-Marchenko) theory and the direct envelope inversion (DEI) using reflection data. The difference between wave equation and Schrödinger equation is that the latter has a potential independent of frequency while the former has a frequency-square dependency in the potential. I also point out that the traditional GLM equation for potential inversion can only recover the high-wavenumber components of impedance profile. I propose to use the Schrödinger impedance equation for direct impedance inversion and introduce a singular impedance function which also corresponds to a singular potential for the reconstruction of impedance profile, including discontinuities and long-wavelength velocity structure. I will review the GLM theory and its application to impedance inversion including some numerical examples. Then I analyze the recently developed multiscale direct envelope inversion (MS-DEI) and its connection to the inverse Schrödinger scattering. It is conceivable that the combination of strong-scattering inversion (inverse Schrödinger scattering) and weak-scattering inversion (local optimization based inversion) may create some inversion methods working for a whole range of inversion problems in geophysical exploration.

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1 Introduction

The earth medium has a very nonlinear response to the excitation of seismic waves. Therefore, the parameter inversion, especially velocity inversion, is expected generally to be nonlinear for seismic exploration. Newton's scheme of local optimization has the assumption of weak nonlinearity so that the nonlinear search can be realized by a local linearization [1]. It is well-known that the local minimization approach has many difficulties, such as the cycle-skipping, local minima, or divergence of the iterative process (for a review, see [2]). On the other hand, we know that Born data has a simple linear relationship with the parameter change no matter how strong the perturbations are. Born scattering observes linear superposition, so the final scattered field is the superposition of all the scattered fields by each element volume of the whole medium. If the data collected are the Born data, everything will be simple. For a full illumination, the inversion problem can be solved by a linear inversion. However, the real scattering process in the earth is nonlinear, and the strong interaction and interference between scattered waves including multiply scattered waves from different elements destroy the linearity of the Born data. For strong-nonlinear data due to strong scattering, the linearity of the data response cannot be recovered by any linear process. This why the Newton type inversion has to start the iterative process in a nearly linear region. That means the iterative process must start with a "good" initial model which is "close" to the true model.

Different from the popular Newton's method, there are some special approaches which can solve strong-scattering inversion without weak-scattering assumption. One approach is the direct nonlinear inversion or nonlinear inverse scattering based on wave equation. Since the Newton's local optimization method uses the linearized sensitivity operator, one approach to deal with strong nonlinearity is to introduce directly the ***nonlinear sensitivity operator*** (*NLSO*) to the inversion. Wu and Zheng [3] proved that the higher-order terms of the NLSO, i.e. the higher order Fréchet derivatives correspond to the higher-order multiple-scattering terms in the scattering series. Scattering series in solving forward and inverse scattering problem has been known for a long time in the literature. However, in the case of strong-scattering for wave equation, the conventional Born-Neumann series may not converge (see e.g., [4]), which is the cause for the divergence or falling into local minima of nonlinear inversion. To overcome the divergence problem of scattering or inverse-scattering series, renormalization or renormalization group (RG) theory/method was applied to the scattering or inverse-scattering series in different ways. One way is to use the Volterra integral to renormalize the scattering series [5,6]. Yao et al. [6] uses a perturbation series approach and calculated the Volterra inverse scattering series (VISS) to the third order, but the convergence of the VISS is not discussed. Weglein et al. [7] proposed to use different task-oriented subseries to mitigate the divergence problem. The other way to renormalize the scattering series is to use the De Wolf series [3,8–11]. Geng et al. [12] has applied the method to inversion using up to the second order NLSO.

Another approach for nonlinear inversion, especially for strong-nonlinear inversion