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Performance Enhancement for High-Order Gas-Kinetic Scheme Based on WENO-Adaptive-Order Reconstruction

Xing Ji[1] and Kun Xu[1,2,*]

[1] *Department of Mathematics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong SAR.*
[2] *Department of Mechanical and Aerospace Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong SAR.*

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Abstract. High-order gas-kinetic scheme (HGKS) has been well-developed in the past years. Abundant numerical tests including hypersonic flow, turbulence, and aeroacoustic problems, have been used to validate its accuracy, efficiency, and robustness. However, there are still rooms for its further improvement. Firstly, the reconstruction in the previous scheme mainly achieves a fifth-order accuracy for the point-wise values at a cell interface due to the use of standard WENO reconstruction, and the slopes of the initial non-equilibrium states have to be reconstructed from the cell interface values and cell averages again. The same order of accuracy for slopes as the original WENO scheme cannot be achieved. At the same time, the equilibrium state in space and time in HGKS has to be reconstructed separately. Secondly, it is complicated to get reconstructed data at Gaussian points from the WENO-type method in high dimensions. For HGKS, besides the point-wise values at the Gaussian points it also requires the slopes in both normal and tangential directions of a cell interface. Thirdly, there exists visible spurious overshoot/undershoot at weak discontinuities from the previous HGKS with the standard WENO reconstruction. In order to overcome these difficulties, in this paper we use an improved reconstruction for HGKS. The WENO with adaptive order (WENO-AO) [2] method is implemented for reconstruction. Equipped with WENO-AO reconstruction, the performance enhancement of HGKS is fully explored. WENO-AO not only provides the interface values, but also the slopes. In other words, a whole polynomial inside each cell is provided by the WENO-AO reconstruction. The available polynomial may not benefit to the high-order schemes based on the Riemann solver, where only points-wise values at the cell interface are needed. But, it can be fully utilized in the HGKS. As a result, the HGKS becomes simpler than the previous one with the direct implementation of cell interface values and their slopes from WENO-AO. The additional reconstruction of equilibrium state at the beginning of each time step can be avoided as well by dynamically merging the reconstructed

*Corresponding author. *Email addresses:* `xjiad@connect.ust.hk` (X. Ji), `makxu@ust.hk` (K. Xu)

non-equilibrium slopes. The new HGKS essentially releases or totally removes the above existing problems in the previous HGKS. The accuracy of the scheme from 1D to 3D from the new HGKS can recover the theoretical order of accuracy of the WENO reconstruction. In the two- and three-dimensional simulations, the new HGKS shows better robustness and efficiency than the previous scheme in all test cases.

AMS subject classifications: 76N15, 35L65, 35Q20, 35Q30, 76M12

Key words: High-order finite volume scheme, gas-kinetic scheme, WENO reconstruction, high-order Navier-Stokes solver.

1 Introduction

The gas-kinetic scheme (GKS) targets on the Euler and Navier-Stokes solutions under the finite volume framework [43]. Its interface flux is based on a time evolution solution of the kinetic model equation, such as the Bhatnagar-Gross-Krook (BGK) model [4]. High-order gas kinetic scheme (HGKS) has been developed systematically in the past decade [20]. In comparison with traditional Riemann solver based high-order CFD methods [38, 39], the distinguishable points of HGKS include the followings: (i) The time evolving gas distribution function at a cell interface provides a multiple scale flow physics from the kinetic particle transport to the hydrodynamic wave propagation, which unifies the evolution from the upwind flux vector splitting to the central difference Lax-Wendroff type discretization. (ii) Both inviscid and viscous fluxes are obtained from the moments of a single time-dependent gas distribution function. (iii) The flux in GKS has the multi-dimensional properties [46], where both normal and tangential derivatives of flow variables around a cell interface contribute the time evolution solution of the gas distribution function. (iv) The time evolving gas distribution function at the cell interface not only provides the flux function, but also the time evolution of macroscopic flow variables. The updated interface flow variables at the beginning of next time step can be directly used to construct higher-order compact schemes [28, 29, 49]. (v) Different from the Runge-Kutta (RK) time discretization for achieving high-order temporal accuracy, the multi-stage multi-derivative (MSMD) method provides a higher-order time evolution solution with less middle stages due to the existence of the time-derivative of the interface flux function in HGKS. Inspired initially by the higher-order generalized Riemann problem [18], a two-stage fourth-order GKS is proposed [31]. (vi) The multi-scale unified GKS (UGKS) for the whole flow regime from rarefied to continuum one have been developed as well [11, 21, 25]. Recently a family of HGKS have been constructed with only two or three stages for a fifth-order time accurate solution [13]. Based on the same fifth-order WENO reconstruction, the performance of HGKS shows great advantages in terms of efficiency, accuracy, and robustness compared with traditional higher-order schemes with Riemann solver and RK time-stepping techniques. Especially, HGKS can capture flow structures, such as shear instabilities, significantly better than the schemes based on the