Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2019-0056

Vol. **28**, No. 2, pp. 621-660
August 2020

Modified Ghost Fluid Method with Axisymmetric Source Correction (MGFM/ASC)

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Received 4 April 2019; Accepted (in revised version) 16 August 2019

Abstract. In this work, we show that the modified ghost fluid method might suffer pressure mismatch at material interfaces and thus leads to inaccurate numerical results when directly applied to long term simulations of multi-medium flow problems with an axisymmetric source term. We disclose the underlying reason and then develop a technique of linear distribution to take into account the effect of the axisymmetric source on the definition of ghost fluid states. In order to faithfully consider the effect of the source term, the interfacial conditions related to derivatives are derived and linear distributions of ghost fluid states are constructed based on a generalized axisymmetric multi-medium Riemann problem. Theoretical analysis and numerical results show that the modified ghost fluid method with axisymmetric source correction (MGFM/ASC) can effectively eliminate the pressure error.

AMS subject classifications: 35L65, 35L67, 65N85, 76T10

Key words: Multi-medium compressible flow, axisymmetric flow, ghost fluid method, modified ghost fluid method, generalized multi-medium Riemann problem.

1 Introduction

The dynamics of compressible multi-medium flows often gives rise to challenging problems in both theory and numerical simulations. The change in equation of state (EOS) is known to cause numerical inaccuracies or oscillations near material interfaces. To overcome those difficulties, various strategies have been pursued in the past two decades with increasing interest [1–9]. Some methods treat material interfaces as distinct sharp

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interfaces by reformulating the problem using a mixture model [1–6]; an artificial EOS is usually required for constituting the mixture cells. This treatment, however, may lead to less-than-faithful capture of discontinuous response and even result in numerical instabilities, if a shock is transmitted across the interface. Comparatively speaking, an immiscible model seems to be more reasonable in the presence of immiscible interfaces. Researchers can take all kinds of effective measures to deal with an immiscible interface, such as volume of fluid method [10], level set technique [11] or front tracking technique [12]. But the interfacial state should be faithfully simulated to suppress any undesired numerical oscillation, especially, when there is a strong nonlinear wave interaction occurring at the interface.

The idea of ghost fluid method (GFM) [13–19] has provided us a simple and flexible way for handling multi-medium flows with immiscible material interfaces. The GFM-based techniques have been applied by many researchers to a wide range of problems. Through specially defining ghost nodes and ghost fluid states, the computation can be carried out as if in a single medium for a GFM-based method. As a result, the numerical schemes for single-medium flows can be employed without any change, leading to its easy extension to multi-dimensions as well.

Variants of GFM in literatures differ in the way how the ghost fluid state is populated. Fedkiw et al. proposed the original GFM (OGFM) [15] by using the local real fluid velocity and pressure to define the corresponding ghost fluid state. Later, the gas-water version GFM (GWGFM) [16], in which the ghost fluid state is defined by employing the velocity from the water (stiff medium) and the pressure from the gas (less stiff medium), was specially presented for coupling non-stiff fluid (gas) and stiff fluid (water). Although the above two GFMs are problem-related and not suitable for some cases like high speed jet/shock impacting [20], the simplicity and the easy employment promote their developments [6,21–24]. In order to take into account the effects of wave interaction and material properties, Liu et al. proposed the original modified GFM (MGFM) [17] by carrying out characteristic analysis on the waves arriving at the interface and solving a local multi-medium Riemann problem. Following the idea of Riemann problem-based technique in [17], the interface interaction GFM (IGFM) [18], the real GFM (RGFM) [19] and the practical GFM (PGFM) [20] have also been developed. The Riemann problem-based technique, discussed also in this paper, is characterized by (approximately) solving a multi-medium Riemann problem to define ghost fluid states. This differs from the OGFM and the GWGFM where ghost fluid states are defined via using the local flow state or extrapolating from the local real fluid. The Riemann problem-based technique has been shown to be robust and less problem-related and successfully applied to solve a wide range of problems involving strong shocks interacting with gas-gas, gas-water and gas-water-solid interfaces [17–21, 25–35]. Furthermore, it has been proved that the error estimate by the MGFM is third-order accurate near the interface for a multi-medium Riemann problem [36, 37].

However, since the effects of source terms are not considered in the definition of ghost fluid states, the conventional MGFM-type methods based on the solution of multi-