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FESTUNG: A MATLAB / GNU Octave Toolbox for the Discontinuous Galerkin Method. Part IV: Generic Problem Framework and Model-Coupling Interface

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Abstract. This is the fourth installment in our series on implementing the discontinuous Galerkin (DG) method as an open source MATLAB / GNU Octave toolbox. Similarly to its predecessors, this part presents new features for application developers employing DG methods and follows our strategy of relying on fully vectorized constructs and supplying a comprehensive documentation. The specific focus of the current work is the newly added generic problem implementation framework and the highly customizable model-coupling interface for multi-domain and multi-physics simulation tools based on this framework. The functionality of the coupling interface in the FESTUNG toolbox is illustrated using a two-way coupled free-surface / groundwater flow system as an example application.

AMS subject classifications: 35L20, 65M60, 68N30, 76B07, 76S05, 97P30

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1 Introduction

The previous papers in the FESTUNG series dealt with the most common differential operators such as the linear diffusion [1] or advection [2] as well as with different types of

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discontinuous Galerkin (DG) discretizations, namely the standard local DG (LDG [1]) or hybridized DG (HDG [3]). In the time span between the previous installment [3] and the current work, several applications involving non-linear equations were implemented using our MATLAB / GNU Octave FESTUNG [4] toolbox; those include the two-dimensional shallow-water equations [5] and mean curvature flow [6, 7]. In addition, a general overview paper documenting the current state of development of FESTUNG has been submitted [8] and is currently under review. The present study, however, sets a much more ambitious goal: Presenting a model-coupling interface that allows to create very complex simulation systems consisting of multiple self-contained simulation tools and interacting by exchanging data usually in the form of solution vectors. The more specific objectives of this work include developing an abstract coupling concept capable of supporting very general types of model coupling as well as implementing and testing this concept in the framework of FESTUNG. Just as in the previous parts of this series, our performance-optimized, fully vectorized implementation with detailed documentation is freely available as an open source software.

Very few physical systems are truly isolated and thus can be modeled in a standalone fashion without the help of some more or less realistic assumptions. Often, the setting can be simplified, and action and reaction of other physical systems can be accounted for in a single-physics model via boundary conditions, forcing terms, parametrizations, etc. However, many important problems do not lend themselves readily to such simplifications; hence the need for coupled models. Examples of widely used multi-physics applications include, in particular, ocean-atmosphere-land systems constituting the staple of climate modeling or fluid-structure interaction models very common in civil engineering and medical applications. This multi-physics label can sometimes be somewhat misleading (see a discussion of this issue in a very comprehensive overview paper [9]) and is also often used to describe interactions between different mathematical representations, or numerical schemes, or grids, or motion scales within the same 'physics'.

For coupled multi-physics problems, one can generally identify three main classes of setups: shared domain/ multiple physics (e.g. coupled subsurface flow/geomechanics), multiple domain/shared physics (e.g. regional grid nesting in ocean or atmosphere simulations), and multiple domain/multiple physics. The first two permit a number of performance-relevant simplifications; although our new problem implementation framework accommodates either, here we focus on the third, the most general type of setup, for which a number of key aspects have to be considered:

- **Modeling issues:** Those include the physical and mathematical consistency questions such as conservation properties, physically meaningful interface conditions, well-posedness of the mathematical problem, etc.
- **Numerical issues:** Different mathematical models often require specialized numerical methods; in addition, computational meshes corresponding to separate physical systems do not necessarily match at the interface (or coincide in the case of a shared domain setup). This gives rise to a number of challenges such as con-