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Time Implicit Unified Gas Kinetic Scheme for 3D Multi-Group Neutron Transport Simulation

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Abstract. In this paper, a time implicit unified gas kinetic scheme (IUGKS) for 3D multi-group neutron transport equation with delayed neutron is developed. The explicit scheme, implicit 1st-order backward Euler scheme, and 2nd-order Crank-Nicholson scheme, become the subsets of the current IUGKS. In neutron transport, the microscopic angular flux and the macroscopic scalar flux are fully coupled in an implicit way with the combination of dual-time step technique for the convergence acceleration of unsteady evolution. In IUGKS, the computational time step is no longer limited by the Courant-Friedrichs-Lewy (CFL) condition, which improves the computational efficiency in both steady and unsteady simulations with a large time step. Mathematically, the current scheme has the asymptotic preserving (AP) property in recovering automatically the diffusion solution in the continuum regime. Since the explicit scanning along neutron traveling direction within the computational domain is not needed in IUGKS, the scheme can be easily extended to multi-dimensional and parallel computations. The numerical tests demonstrate that the IUGKS has high computational efficiency, high accuracy, and strong robustness when compared with other schemes, such as the explicit UGKS, the commonly used finite difference, and finite volume methods. This study shows that the IUGKS can be used faithfully to study neutron transport in practical engineering applications.

AMS subject classifications: 82D75, 82C70, 82C40

Key words: UGKS, neutron transport, time implicit scheme.

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1 Introduction

In this paper, an implicit unified gas kinetic scheme (IUGKS) will be developed for high efficiency 3D multi-group neutron transport simulation. The neutron transport is the type of Boltzmann equation [1]. The equation describes the evolution of the neutron angular flux ϕ in a high-dimensional phase space $(\mathbf{x},t,\mathbf{\Omega},E)$, where $\mathbf{x}$ is the position, t is the time variable, $\mathbf{\Omega}$ is the traveling direction of neutron, and E is the neutron energy.

For the neutron transport, the value of the cross section determines different transport regimes. Traditional numerical methods use decoupled numerical discretizations for the neutron free transport and collision. In order to get a physically valid solution, the spatial and temporal resolutions in these schemes are limited by the neutron mean free path and collision time, respectively. When the cross section is large, the dynamics of neutron transport is described by the diffusion equation. In order to resolve the intensive collisions, the cell size and time step become very small and the computation cost in the operator splitting method is extremely high. To release the stiffness and accelerate convergence of transport problems in diffusion-like regions, the synthetic scheme is the most popular method [2, 3]. The main idea of the synthetic method is to accelerate convergence of source iteration [3] by adding an auxiliary low-order approximation equation to the original transport equation. In recent studies, the method has been widely used in neutron transport [4], radiation transport [5] and the rarefied gas dynamics (RGD) simulations [6]. However, the synthetic acceleration method has not been maturely used in the time-dependent simulations yet.

Considering the multi-scale nature of neutron transport and its diffusive regime [7–9], an asymptotic preserving (AP) IUGKS will be developed for the study of neutron transport. When holding fixed mesh size and time step and letting the Knudsen number go to zero, the AP scheme should automatically recover the diffusion limit solution [7,10]. The early UGKS [11–13] has been developed for RGD. Great successes has been achieved for simulating flows in all regimes uniformly. The scheme has been extended to solve the general linear kinetic model, i.e., the purely radiative transport by Mieussens [14] and the coupled system of radiative transport and energy exchange with background material by Sun et al. [15, 16]. In UGKS, the particles' transport and collision are fully coupled in a time-dependent evolution solution at a cell interface. The ratio of the time step to the particle collision time determines the transport mechanism, such as particle free transport or diffusive wave propagation. As a result, in order to capture physics in different regimes the temporal and spatial resolutions in UGKS are not limited by the neutron mean free path and collision time, respectively. The scheme can solve the transport problem efficiently when the neutron transport passes through materials in different regimes with a large variation of cross section.

The UGKS is an explicit scheme and the time step is limited by the CFL condition. The computational cost is very high in simulations once the time step is limited by the smallest cell in the computational domain. In the RGD and radiative transfer simulations, the time implicit schemes for UGKS have been developed and used in practical