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High-Order Gas-Kinetic Scheme in Curvilinear Coordinates for the Euler and Navier-Stokes Solutions

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Abstract. The high-order gas-kinetic scheme (HGKS) has achieved success in simulating compressible flows with Cartesian meshes. To study the flow problems in general geometries, such as the flow over a wing-body, the development of HGKS in general curvilinear coordinates becomes necessary. In this paper, a two-stage fourth-order gas-kinetic scheme is developed for the Euler and Navier-Stokes solutions in the curvilinear coordinates from one-dimensional to three-dimensional computations. Based on the coordinate transformation, the kinetic equation is transformed first to the computational space, and the flux function in the gas-kinetic scheme is obtained there and is transformed back to the physical domain for the update of flow variables inside each control volume. To achieve the expected order of accuracy, the dimension-by-dimension reconstruction based on the WENO scheme is adopted in the computational domain, where the reconstructed variables are the cell averaged Jacobian and the Jacobian-weighted conservative variables. In the two-stage fourth-order gas-kinetic scheme, the point values as well as the spatial derivatives of conservative variables at Gaussian quadrature points have to be used in the evaluation of the time dependent flux function. The point-wise conservative variables are obtained by ratio of the above reconstructed data, and the spatial derivatives are reconstructed through orthogonalization in physical space and chain rule. A variety of numerical examples from the accuracy tests to the solutions with strong discontinuities are presented to validate the accuracy and robustness of the current scheme for both inviscid and viscous flows. The precise satisfaction of the geometrical conservation law in non-orthogonal mesh is also demonstrated through the numerical example.

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1 Introduction

In recent decades, there have been continuous interests and efforts on the development of high-order schemes. With the development of computational aero-acoustics (CAA), large eddy simulations (LES), and direct numerical simulations (DNS), the construction of high-order numerical scheme becomes extremely demanding, and many high-order finite volume schemes on unstructured meshes have been proposed for the complicated geometries [1, 17, 20]. However, the direct implementation in the physical space brings big challenges. The complexity of algorithms and codes increases dramatically because of the difficulty in choosing stencils, especially in the multi-dimensional reconstruction. To overcome the drawback, an efficient way is to apply the finite volume method in the curvilinear coordinate system, where the structured meshes are used. The technique of curvilinear or mapped coordinates is widely used in engineering [15, 44]. In principle, given a suitable mapping function, any problem defined on a general physical domain can be transformed into a computational domain which is equidistant and Cartesian. Although the flexibility may be reduced in comparison with unstructured meshes, the good numerical characteristics are preserved. The first one is the exact global conservation property, which is only approximately satisfied in the high-order finite difference method [27], and the second one is the strict adherence to the integral form for numerical simulations [14]. Furthermore, the standard numerical schemes on the Cartesian and equidistant grids can be used [40].

In the past decades, the gas-kinetic scheme (GKS) based on the Bhatnagar-Gross-Krook (BGK) model [4, 9] has been developed systematically for the computations from low speed flows to supersonic flows [48, 49]. Different from the traditional finite volume and finite difference schemes [41, 43], GKS presents a gas evolution process from kinetic scale to hydrodynamic scale, where both inviscid and viscous fluxes are recovered from a multidimensional time-dependent gas distribution function. Based on the unified coordinate transformation [21], the second-order gas-kinetic scheme was developed under the moving-mesh framework as well [24, 25]. The flux evaluation in the GKS is based on the time evolution of flow variables from an initial piece-wise discontinuous polynomials around each cell interface. Thus, the high-order spatial and temporal evolutions of a gas distribution function are coupled nonlinearly. With the spatial and temporal coupled gas distribution function, the one-stage third-order GKS was developed [30, 33]. In comparison with other high-order schemes with Riemann flux [41], it integrates the flux function over a time step analytically without employing the multi-stage Runge-Kutta techniques [18]. However, with the one-stage gas evolution model, the formulation of GKS can become very complicated for the further improvement, such as the one-stage fourth-