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Magnetic Deformation Theory of a Vesicle

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Abstract. We have extended the Helfrich's spontaneous curvature model [M. Iwamoto and Z. C. Ou-Yang. Chem. Phys. Lett. **590**(2013)183; Y. X. Deng, et al., EPL. **123**(2018)68002] of the equilibrium vesicle shapes by adding the interaction between magnetic field and the constituent molecules to explain the phenomena of the reversibly deformation of artificial stomatocyte [P. G. van Rhee, et al., Nat. Commun. **Sep 24;5:5010**(2014), doi: 10.1038/ncomms6010] and the anharmonic deformation of a self-assembled nanocapsules of bola-amphiphilic molecules and the linear birefringence [O.V. Manyuhina, et al., Phys. Rev. Lett. **98**(2007)146101]. However, the sophisticated mathematics in differential geometry is still covered. Here, we present the derivations of formulas in detailed to reveal the perturbation of deformation ψ under two cases. New features such as the influence of temperature on the bend modulus of vesicle membrane have been revealed.

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Key words: Spontaneous curvature model, deformation of vesicle, magnetic interaction, variation.

1 Introduction

The spontaneous curvature model [1] of the equilibrium shapes and deformations of lipid bilayer vesicles, which has been proposed by Helfrich for more than four decades, was used to successfully explain the biconcave discoid shape of red blood cells [2–5], so that it is well accepted in biophysics [6]. Particularly, it predicted that the anchor rings generates circles of radii in the ratio of $1/\sqrt{2}$ [7], and the ratio was precisely confirmed by experiments in toroidal vesicles [8], phospholipid membrane [9] and micelles [10]. Recently, the curvature elasticity model has been extended to investigate shapes in soft matter,

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such as the helical structures in carbon nanotubes [11] and in bile ribbons [12], cylindrical structures in the smectic-A phase [13] and in peptide nanotubes [14], the circle-domain instability in lipid monolayers [15] and icosahedral structures in virus capsids [16,17].

If a vesicle is assembled from diamagnetic amphiphilic block-copolymers with a highly anisotropic magnetic susceptibility, we can manipulate its deformation by an external magnetic field. For example, the artificial stomatocyte [18] can be reversibly opened and closed by varying an external magnetic field. The artificial stomatocyte, thus, has a great potential to transport drug to a target. On the other hand, the small deformation can be measured by birefringence [19]. The magnetic deformation theory had been proposed by adding the interaction between the magnetic field and the constituent molecules into the shape energy [20,21], the experimental data were explained satisfactorily.

However, the mathematics in differential geometry is still covered. Here, we present the derivations of formulas in detailed to reveal the perturbation of deformation ψ under two cases.

2 Free energy of a vesicle in magnetic field

Physically, the shape of the vesicle is finally determined by the equilibrium state, at which the energy of any physical system must be at its minimum, i.e. the equilibrium energy of a vesicle must be less than that of other deformation induced by a slightly perturbation. Helfrich proposed that the shape energy of a vesicle can be given by

$$F_1 \equiv \frac{1}{2}\kappa_c \oint (2H+c_0)^2 \mathrm{d}A + \Delta p \int \mathrm{d}V + \lambda \oint \mathrm{d}A, \tag{2.1}$$

where κ_c is the bend modulus of vesicle membrane, $H \equiv -(c_1+c_2)/2$ is the mean value of the two principal curvatures(c_1,c_2), c_0 is the spontaneous curvature, $\Delta p \equiv p_{\text{out}} - p_{\text{in}}$ is the difference pressure of transmembrane, λ is the tensile stress acting on the membrane. V and A are the volume and the surface area of the vesicle respectively. Mathematically, Δp and λ may be considered as Lagrange multipliers.

If the vesicle is assembled from diamagnetic amphiphilic block-copolymers with a highly anisotropic magnetic susceptibility and is in a magnetic field, the interaction between the magnetic field ($\vec{\boldsymbol{\mathcal{H}}}$) and the constituent molecules($F_{\text{B}} \equiv -\frac{1}{2}\Delta\chi t\mu \oint (\vec{\boldsymbol{\mathcal{H}}}\cdot\vec{\boldsymbol{n}})^2 \mathrm{d}A$) has to be added into the shape energy

$$\begin{aligned} F &\equiv F_1 + F_{\text{B}} \\ &= \frac{\kappa_c}{2}\oint (2H+c_0)^2 \mathrm{d}A + \Delta p \int \mathrm{d}V + \lambda \oint \mathrm{d}A - \frac{1}{2}\Delta\chi t\mu \oint \left(\vec{\boldsymbol{\mathcal{H}}}\cdot\vec{\boldsymbol{n}}\right)^2 \mathrm{d}A, \end{aligned} \tag{2.2}$$

where t is the thickness of the membrane of vesicle, μ is the magnetic permeability, $\vec{\boldsymbol{n}}$ is the outward unit normal and $\Delta\chi \equiv \chi_{\parallel} - \chi_{\perp}$, in which χ is the diamagnetic susceptibility, while $\chi_{\parallel}$ and $\chi_{\perp}$ are diamagnetic susceptibility parallel and perpendicular to $\vec{\boldsymbol{n}}$ respectively.