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A High-Order Cell-Centered Discontinuous Galerkin Multi-Material Arbitrary Lagrangian-Eulerian Method

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Abstract. In this paper, a high-order cell-centered discontinuous Galerkin (DG) multi-material arbitrary Lagrangian-Eulerian (MMALE) method is developed for compressible fluid dynamics. The MMALE method utilizes moment-of-fluid (MOF) interface reconstruction technology to simulate multi-materials of immiscible fluids. It is an explicit time-marching Lagrangian plus remap type. In the Lagrangian phase, an updated high-order discontinuous Galerkin Lagrangian method is applied for the discretization of hydrodynamic equations, and Tipton's pressure relaxation closure model is used in the mixed cells. A robust moment-of-fluid interface reconstruction algorithm is used to provide the information of the material interfaces for remapping. In the rezoning phase, Knupp's algorithm is used for mesh smoothing. For the remapping phase, a high-order accurate remapping method of the cell-intersection-based type is proposed. It can be divided into four stages: polynomial reconstruction, polygon intersection, integration, and detection of problematic cells and limiting. Polygon intersection is based on the "clipping and projecting" algorithm, and detection of problematic cells depends on a troubled cell marker, and a *posteriori* multi-dimensional optimal order detection (MOOD) limiting strategy is used for limiting. Numerical tests are given to demonstrate the robustness and accuracy of our method.

AMS subject classifications: 65M60

Key words: Multi-material ALE, discontinuous Galerkin, moment-of-fluid, Tipton's pressure relaxation closure model, a *posteriori* MOOD limiting strategy.

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1 Introduction

Simulations of compressible multi-material fluid flows can be classified into two computational frameworks, each with their own advantages and disadvantages. The first is Lagrangian, in which the mesh is embedded in the fluid and moves with it; the second, known as Eulerian, treats the mesh as a fixed reference frame through which the fluid moves. In order to combine the advantages of the Lagrangian and Eulerian methods, Hirt et al. [1] proposed an arbitrary Lagrangian-Eulerian (ALE) method in which the grid points may be moved in some arbitrarily specified ways. Most ALE algorithms [2–7] consist of three phases, a Lagrangian phase, a rezoning phase, and a remapping phase.

The traditional ALE method uses cell edges to explicitly describe the material interfaces, which requires that each cell contains only one material. When the material interface deforms severely, it is very difficult to generate a new mesh with good quality. In some problems involving strong shearing deformation, it is even impossible to perform rezoning successfully. In order to deal with this problem, Peery et al. [8] proposed the multi-material ALE method (MMALE) by combining the traditional ALE method with the interface capture method. This method allows for multiple materials in a single cell and therefore affords additional flexibility over the traditional ALE method. With these flexibility, the MMALE method can accurately simulate problems involving strong shearing deformation, which are difficult to deal with by the traditional ALE method.

The major process of the MMALE method is as follows. Firstly, the thermodynamic states of each material and the computational mesh are updated in the Lagrangian phase. Unlike the traditional ALE method, a thermodynamic closure model is required in the mixed cells to define how the volume fraction and state of the individual materials evolve during the Lagrangian phase. When the mesh and the material interface deform severely, an interface reconstruction method is utilized to reconstruct interfaces in the deformed Lagrangian cells to provide information of the interfaces for the latter remapping phase. Then in the rezoning phase, a new mesh is generated which is not demanded for material-interface-fitting, so that good geometric quality can be achieved with severe interface deformation. Finally, in the remapping phase, variables such as density, momentum, energy, volume fractions and material centroids are remapped from the deformed Lagrangian mesh to the rezoned one.

The most popular closure model of the MMALE method is Tipton's pressure relaxation closure model [9, 10]. In this model, a relaxation mechanism like viscosity is introduced to make the pressures of materials within a mixed cell move toward pressure equilibration. This model invokes conservation of volume and some form of conservation of total internal energy. In addition, it is assumed that the flow is isentropic when the model is closed. Tipton's model does not require information about the material interface.

The interface reconstruction methods generally used in MMALE methods are volume of fluid (VOF) [11–13] and MOF methods [14–16]. The VOF methods first use the volume fractions from neighboring cells to determine the normal vector of the interface and then