A CFD-Aided Galerkin Method for Global Linear Instability Analysis

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Abstract. Global linear instability analysis is a powerful tool for the complex flow diagnosis. However, the methods used in the past would generally suffer from some disadvantages, either the excessive computational resources for the low-order methods or the tedious mathematical derivations for the high-order methods. The present work proposed a CFD-aided Galerkin methodology which combines the merits from both the low-order and high-order methods, where the expansion on proper basis functions is preserved to ensure a small matrix size, while the differentials, incompressibility constraints and boundary conditions are realized by applying the low-order linearized Navier-Stokes equation solvers on the basis functions on a fine grid. Several test cases have shown that the new method can get satisfactory results for one-dimensional, two-dimensional and three-dimensional flow problems and also for the problems with complex geometries and boundary conditions.

1 Introduction

Flow instability analysis plays a key role in understanding the origin of many complex fluid motions including turbulence. Among all kinds of instability analysis methods,

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linear instability analysis is the simplest and most widely used one, and it can obtain satisfactory results in shear flows, rotating flows, buoyancy-driven convections and surface-tension-driven instability with steady and laminar basic state. The classic Tollmien's theory and Orr-Sommerfeld equation can deal with the linear instability of one-dimensional parallel flow [1–4]. Consideration of another slowly evolving spatial direction will result in parabolised stability equations [5–7]. However, when the base flow is strongly inhomogeneous in more than one direction, the general global instability should be taken into account and the governing equations are no longer one-dimensional, raising the challenges on the establishment and the solution of respective eigenvalue problems.

Pierrehumbert and Widnall (1982) [8] published the pioneering work on global linear instability and discussed the instability of shear-layer vortices using spectral collocation method. Since then, the spectral collocation method and the spectral Galerkin method have been widely used [9, 10] for simple flow problems due to the small matrix size which benefits from the high-order accuracy of the methods. For piecewise regular domains such as backward facing step, multi-domain technique can be applied [11]. For more general geometry, spectral element method [12] and spectral/hp method [13] can be introduced as the generalization of multi-domain spectral method. Many other investigations using spectral method can be seen in [7, 14–22]. Although global spectral methods are accurate and more likely to provide a small matrix for the full eigen-spectrum computation, the mathematical derivation is often tedious, as can be seen in [17].

Due to the simplicity and flexibility, simple discretization methods, including the finite difference method [23–28], the finite element method [29–31], the finite volume method [25, 32, 33] and the lattice Boltzmann method [34, 35], have also been used to study the global linear instability. In order to make the analysis accurate enough, the simple low-order discretization methods would require large matrix size to fully characterize the linear operator. With high-order finite difference schemes, the required matrix size can be reduced [36], but it is still larger than those obtained in high-order spectral methods.

Although matrix-free subspace iteration approach [37] can be applied to obtain the leading eigen-modes, the computation expenses will increase heavily if more high-accuracy modes are required. Therefore, matrix-forming approach is still the most reasonable choice on condition that the matrix size is as small as possible. As suggested by Theofilis [38], full eigenspectrum should be derived if possible because it is helpful for a deep understanding of the problem. In fact, there are still some interesting problems with simple geometry which can be analyzed with spectral methods (smallest matrix size required), but it is hard to realize due to the extremely tedious derivations.

The present work is devoted to finding a new way to construct the eigenproblem matrix which represents the linear operator. In the newly proposed method, the matrix is formed with the help of globally smooth basis functions while the differential operations are realized with the aid of the linearized CFD code. The remainder of the paper is organized as follows. Section 2 introduces the general problem setup and numerical method in detail. The new method will be validated in Section 3 where several one-dimensional