Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2019-0184

Vol. **29**, No. 1, pp. 148-185
January 2021

Multi-Mesh-Scale Approximation of Thin Geophysical Mass Flows on Complex Topographies

Yih-Chin Tai[1,*], Jeaniffer Vides[2], Boniface Nkonga[3] and Chih-Yu Kuo[4]

[1] *Department of Hydraulic and Ocean Engineering, National Cheng Kung University, Tainan, Taiwan.*
[2] *LEMMA R&D Services, Sophia Antipolis, Biot, France & Maison de la Simulation, Gif-sur-Yvette, France.*
[3] *Université Côte d'Azur (UCA), LJAD Nice & Inria Sophia Antipolis – Méditerranée, France*
[4] *Research Center for Applied Sciences, Academia Sinica, Taipei, Taiwan.*

Received 23 October 2019; Accepted (in revised version) 16 May 2020

Abstract. This paper is devoted to a multi-mesh-scale approach for describing the dynamic behaviors of thin geophysical mass flows on complex topographies. Because the topographic surfaces are generally non-trivially curved, we introduce an appropriate local coordinate system for describing the flow behaviors in an efficient way. The complex surfaces are supposed to be composed of a finite number of triangle elements. Due to the unequal orientation of the triangular elements, the distinct flux directions add to the complexity of solving the Riemann problems at the boundaries of the triangular elements. Hence, a vertex-centered cell system is introduced for computing the evolution of the physical quantities, where the cell boundaries lie within the triangles and the conventional Riemann solvers can be applied. Consequently, there are two mesh scales: the element scale for the local topographic mapping and the vertex-centered cell scale for the evolution of the physical quantities. The final scheme is completed by employing the HLL-approach for computing the numerical flux at the interfaces. Three numerical examples and one application to a large-scale landslide are conducted to examine the performance of the proposed approach as well as to illustrate its capability in describing the shallow flows on complex topographies.

AMS subject classifications: 35L65, 76M12, 58J90, 86-08

Key words: Multi-mesh-scale approach, complex topography, shallow flows, unstructured mesh, vertex-centered formulation.

*Corresponding author. *Email addresses:* yctai@mail.ncku.edu.tw (Y.-C. Tai), jeaniffer.vides@lemma-ing.com (J. Vides), Boniface.Nkonga@unice.fr (B. Nkonga), cykuo06@gate.sinica.edu.tw (C.-Y. Kuo)

1 Introduction

Most geophysical hazardous flows take place over complex topographies. The occurrences of avalanches, landslides or debris flows over non-trivial topographies in mountainous areas are some clear examples. Not surprisingly, there is a strong link between the particular flow path and the geometry of the underlying basal topography. In addition, these flows are generally "thin" (in depth) compared with the large extension in their tangential direction, so that the term "shallow flow" is employed here to characterize thin flows on curved surfaces. For this type of flows, it is customary to make use of a depth-integration process together with the shallowness assumption to asymptotically derive reduced models for the evolution of the depth-averaged velocity and the thickness of the flow. Conventionally, these models are given in Cartesian coordinates, where the topography is added on a horizontal/inclined plane, or in the so-called "locally inclined Cartesian-type" coordinates [10]. With this approach, many successful models with variant rheological relations, with single-phase [1,2], quasi two-phase [3–7], two- or multi-phase (solid, fine solid and fluid) concepts [8–12], have been proposed and have made fruitful achievements. In addition to the ease of imposing constitutive relations, one of the advantages of applying this geometrically simple coordinate system is the less complicated process of its numerical implementation, so that numerical applications are mostly built based on models with this approach. For example, an advanced GIS-supported open-source computational tool, r.avaflow [13–15], has been developed in recent years for describing two-phase (solid and fluid) or multi-phase (solid, fine solid and fluid) flows over complex topographies with respect to the digital elevation map (DEM) by applying the models proposed by [10] and [12], respectively. However, as the depth-averaged models generally employ the shallowness assumption, in which the computed velocities are parallel to the corresponding coordinate axes instead of the exact basal surface, a high variation in topography leads to significant deviations in representing the depth-averaged velocity. To correct this shortcoming, Savage and Hutter [16] first applied a 2D curvilinear coordinate system aligned with the curved bed for modeling granular flows. This 2D curvilinear coordinate system is extended to 3D by introducing a simple reference surface curved in the down-slope direction, and the complex topography is described by superposing a shallow basal topography on it [17–20]. Although the concept of a simple curved reference plane has also been widely adopted, it suffers from the limitation of the predefined unique down-slope direction of the simple curved reference surface, especially in its application to a snaking canyon terrain. Pudasaini and Hutter [21] improved it by imposing a curved and twisted coordinate system, in which the thalweg coincides with the projection of the master curve on a reference surface. The models presented by curved and twisted coordinate system are advanced and rather successful for the flows over generally curved channels [21–23]. However, determining the necessary master curve(s) and the associated reference surface(s) for complex canyon terrains is not always straightforward.

Bouchut and Westdickenberg (BW) [24] proposed the shallow water equations in a